MATH 180A HOMEWORK 3

WINTER 2023

Due date: **Friday 2/10/2023 11:59 PM** (via Gradescope)

1. (*ASV**, *Exercise 2.18*) - 2 points.
   We choose a number from the set $\{10, 11, 12, \ldots, 99\}$ uniformly at random.
   (a) Let X be the first digit and Y the second digit of the chosen number. Show that X and Y are independent random variables.
   (b) Let X be the first digit of the chosen number and Z the sum of the two digits. Show that X and Z are not independent.

2. (*ASV, Exercise 1.36*) - 2 points.
   (a) Let (X, Y) denote a uniformly chosen random point inside the unit square
   $$[0,1]^2 = [0,1] \times [0,1] = \{(x,y) : 0 \leq x, y \leq 1\}.$$
   Let $0 \leq a < b \leq 1$. Find the probability $\mathbb{P}(a < X < b)$, that is, the probability that the x-coordinate X of the chosen point lies in the interval (a, b).
   (b) What is the probability $\mathbb{P}(|X - Y| \leq 1/4)$?

3. (*ASV, Exercise 3.20*) - 2 points.
   Let $c > 0$ and $X \sim \text{Unif}[0, c]$. Show that the random variable $Y = c - X$ has the same cumulative distribution function as X and hence also the same density function.

4. (*ASV, Exercise 3.39*) - 2 points.
   Parts (a) and (b) ask for an example of a random variable X whose cumulative distribution function $F(x)$ satisfies $F(1) = 1/3$, $F(2) = 3/4$, and $F(3) = 1$.
   (a) Make X discrete and give its probability mass function.
   (b) Make X continuous and give its probability density function.

5. (*ASV, Exercise 3.41*) - 3 points.
   We produce a random real number X through the following two-stage experiment. First roll a fair die to get an outcome Y in the set $\{1, 2, \ldots, 6\}$. Then, if $Y = k$, choose X uniformly in the interval $(0, k]$. Find the cumulative distribution function $F(s)$ and the probability density function $f(s)$ of X for $3 < s < 4$.

6. (*ASV, Exercise 3.46*) - 3 points.
   A stick of length ℓ is broken at a uniformly chosen random location. We denote the length of the smaller piece by X.
   (a) Find the cumulative distribution function of X.
   (b) Find the probability density function of X.

**Introduction to Probability*, by David F. Anderson, Timo Seppäläinen, and Benedek Valkó

7. (*ASV, Exercise 2.20*) - 3 points.
   A fair die is rolled repeatedly. Use precise notation of probabilities of events and random variables for the solutions to the questions below.
   (a) Write down a precise sum expression for the probability that the first five rolls give a three at most two times.
   (b) Calculate the probability that the first three does not appear before the fifth roll.
   (c) Calculate the probability that the first three appears before the twentieth roll but not before the fifth roll.

8. (*ASV, Exercise 2.21*) - 3 points.
   Jane must get at least three of the four problems on the exam correct to get an A. She has been able to do 80% of the problems on old exams, so she assumes that the probability she gets any problem correct is 0.8. She also assumes that the results on different problems are independent.
   (a) What is the probability she gets an A?
   (b) If she gets the first problem correct, what is the probability she gets an A?