MATH 180A (Lecture A00)

mathweb.ucsd.edu/~ynemish/teaching/180a

Today: Definition of probability

Next: ASV 1.2

Week 1:

- check the course website
- homework 1 (due Friday, January 20)
- join Piazza

Probability theory

The goal of probability theory is to build mathematical models of experiments with random outcomes

Random outcome = impossible to be predicted with

certainty

1654: starting point, mathematical treatment of gambling problems (Fermat, Pascal)

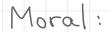
1933: modern rigorous foundation of probability

theory (Kolmogorov)

Warm-up problem

What is the probability that there are at least two student in this room having birthday on the same day (MMIDD)?

100 students, 365 possible birthday dates



Axioms of probability

How to construct a mathematical model of an

experiment with random outcome?

Def. Probability space is the triple (Ω, J, P) , where

· Ω is the

; we call it the sample space

• J is a

(i)

(11)

(iii)

. P is a function that assigns to each event a real

number and satisfies the following properties:



C.

We call function P that satisfies properties (i)-(iii)

, or simply probability.

Example 1: Tossing a coin.

 $\Omega = , J = , P(\varphi) = , P(\Omega) =$ $P(\{H\}) = , P(\{T\}) =$

{H} and {T} are disjoint, therefore, by (iii) $P({H}) + P({T}) = P({H} \cup {T}) =$

For any de [0,1] we have a different probability

measure on I and F.

Fair coin: $P(\{H\}) = P(\{T\}) =$

Examples

Example 2: rolling a fair die

 $= E_1 = \Omega$

 $P({1}) = P({2}) = \cdots = P({6}) = \cdots$

What about the events? Take A={2,4,6}C I.

P(A) =

 $B = \{2, 3, 5\} : P(B) =$

 $C = \{3, 6\}: P(C) =$

P(AUB) =

 $P(B\cap C) =$

B = (, =

A =

Repeated experiments

- What is the sample space if we toss the coin twice ? The outcome is a pair with The collection of such pairs is called the Cartesian product of {H,T} and {H,T}, denoted {H,T} × {H,T} {H,T} × {H,T} = { (H,H), (H,T), (T,H), (T,T) } ← 1 sample space
- More generally, for any sets $\Omega_1, \Omega_2, \ldots, \Omega_k$
 - sample space if we perform experiment 1 with s.s. Di experiment 2 with s.s. Di experiment k with s.s. Di

Finite sample space

Consider a special case when $\#\Omega \times \infty$. Then

 $\Omega = - for n = \# \Omega$

Any event ACD is a finite union of {wil.

The singleton sets {wiy,..., {wny are disjoint.

Therefore, if $A = \{a_1, ..., a_k\}$ for some $a_i \in \Omega$, then

P(A) =

What if additionally we have that $P(\{w_i\}) = \dots = P(\{w_n\})$?

Uniform probability measure and random sampling

If Ω is finite, the uniform probability measure is defined by the following property:

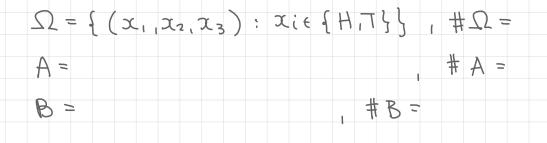
for each $\omega \in \Omega$, $P(\{\omega\}) =$

From (x) this implies that

for any event A, P(A) =

This means that for such models calculating probabilities is reduced to counting. Example Roll a fair dice twice. What is the probability that the sum is 4? $\Omega = \{(i,j): 1 \le i, j \le 6\}$, $A = \{(1,3), (2,2), (3,1)\}, P(A) =$ Uniform probability measure and random sampling

Example A fair coin is tossed 3 times.



$$P(A) = , P(B) =$$