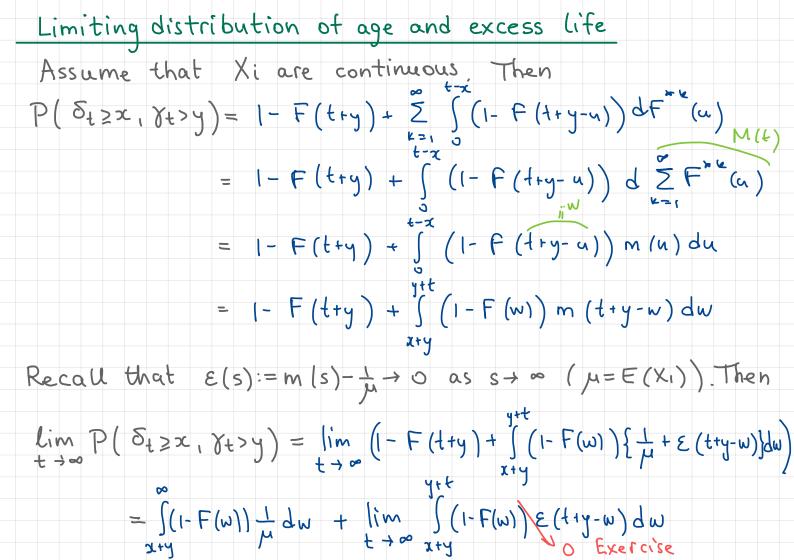
MATH180C: Introduction to Stochastic Processes II https://mathweb.ucsd.edu/~ynemish/teaching/180c Today: Asymptotic behavior of renewal processes Next: PK 7.5, Durrett 3.1, 3.3 Week 6:

HW5 due Friday, May 12 on Gradescope



Joint/limiting distribution of $(\chi_{\ell}, \delta_{\ell})$ Thm. Let F(t) be the c.d.f. of the interrenewal times. Then (a) $P(\chi_{\ell}, y, \delta_{\ell}, \delta_{\ell}) = 1 - F(t+y) + \sum_{k=1}^{\infty} \int_{0}^{t-x} (1 - F(t+y-u)) dF^{*k}(u)$ $t - x = 1 - F(t+y) + \int_{0}^{t-x} (1 - F(t+y-u)) dM(u)$

(b) if additionally the interrenewal times are continuous, $\lim_{t \to \infty} P(\gamma_{t} > \gamma_{1}, \delta_{t} \ge \chi) = \frac{1}{\mu} \int_{\chi_{t} \gamma} (1 - F(\omega)) d\omega \quad (\star)$

If we denote by (yoo, So) a pair of r.v.s with distribution (*)

then yoo and to are continuous r.v.s with densities

 $f_{\gamma\infty}(x) = f_{\varepsilon\infty}(x) = \bot (I - F(x))$

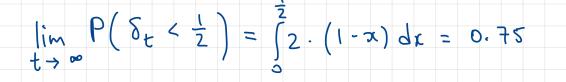
Example

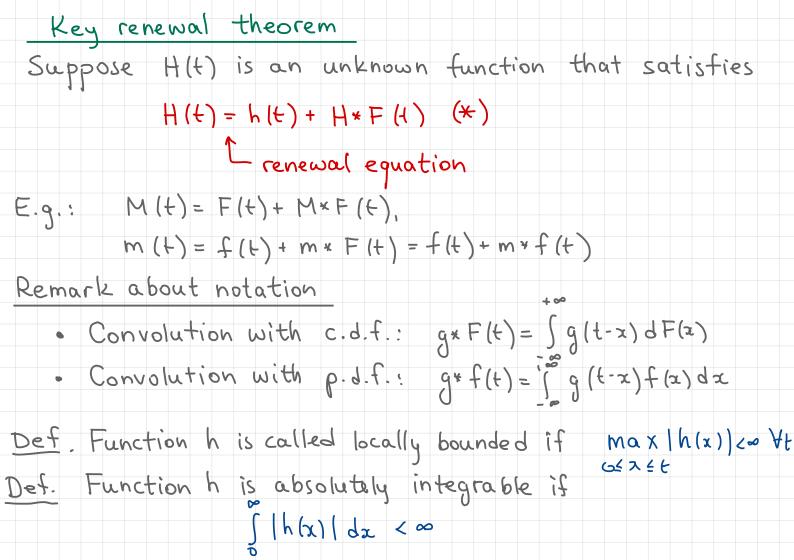
Renewal process (counting earthquakes in California) has interrenewal times uniformly distributed on [0,1] (years). (a) What is the long-run probability that an earthquake will hit California within 6 months?

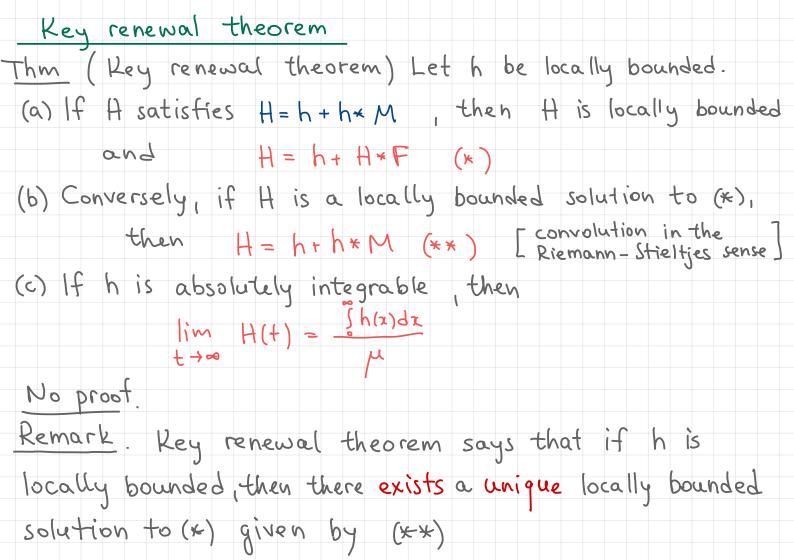
 $\lim_{t \to \infty} P\left(\chi_{t} < \frac{1}{2}\right) = \int_{0}^{2} 2 \cdot (1 - \chi) d\chi = 1 - \chi_{0}^{2} |_{0}^{\chi_{2}} = \frac{3}{4} = 0.75$

(b) What is the long-run probability that it has been

at most 6 months since the last earthquake?







Examples

· Renewal function: M(t) satisfies

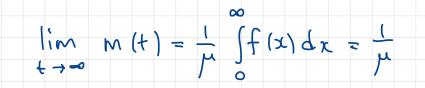
and $M(t) = F(t) + M \cdot F(t) = F(t) + F \cdot M(t)$ $H(t) + h(t) + h(t) + h(t) + h(t) + h(t) + h \cdot M(t)$ F(t) is nondecreasing (so (c) does not apply to

- the renewal equation for M(t)
- Renewal density: m(f) satisfies

m(t) = f(t) + m + F(t)

and = f(t) + f * M(t) (in the Riemann - Stieltjes sense)

f is absolutely integrable, Sfardx = 1, so



Important remark

Let
$$W = (W_1, W_2, ...)$$
 be renewal times of a renewal process,
and denote $W' = (W_1', W_2', ...)$ with
 $W'_i = W_{i+1} - W_1 = X_2 + X_3 + \dots + X_{i+1}$,

shifted cerrival times.

Then:

- W' is independent of W,=X,
- · W' has the same distribution as W

Example

Example. Compute lim $E(\gamma_t)$. Take $H(t) = E(\gamma_t)$

If $X_1 > t$, then $\gamma_t = X_1 - t$; if $X_1 \times t$ condition on $X_1 = s$

 $E(\gamma_t) = E(\gamma_t 1_{X_1 > t}) + E(\gamma_t 1_{X_1 \le t})$

E () + 1 x, +)=

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