

Research Statement

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1 Introduction

I am currently a postdoc in the Mathematics Department of the University of California San Diego, supervised by Prof. Li-Tien Cheng and Prof. Bo Li. I joined the department right after my graduation from Iowa State University with a Ph.D. Degree in Applied Mathematics in May 2008, advised by Prof. Hailiang Liu.

My research interest lies in computational mathematics, applied mathematics and mathematical biology, particularly in level set method and scientific computation. My Ph.D. thesis focuses on the computation of high frequency wave propagation using level set method. This includes computation and analysis of the Euler–Poisson equations, the Schrödinger equation, etc. My current research focuses on the application and development of level set methods in variational implicit solvation model with electrostatics. This project involves the development of numerical tools and collaboration with physicists and biochemists. Working on a biological problem as mathematician, I realize that the interdisciplinary research offers great opportunities for both mathematicians and physicists in better understanding and solving the problems and it is necessary to develop a profound numerical and mathematical tool to facilitate the description of the problems arising in interdisciplinary studies. I also pursue my personal interests in Poisson–Nernst–Planck model from biophysical process and two phase fluid-particle flows, etc. My research experience, current projects and research plans are briefly described below.

2 Research Experience

2.1 A Field Space Based Level Set Method for Computing Multi-valued Solutions to 1D Euler–Poisson Equations

We present a field space based level set method for computing multi-valued solutions to one-dimensional Euler–Poisson equations. The system of these equations has many applications, and in particular arises in semiclassical approximations of the Schrödinger–Poisson equation. The proposed approach involves an implicit Eulerian formulation in an augmented space—called field space, which incorporates both velocity and electric fields into the configuration. Both velocity and electric fields are captured through common zeros of two level set functions, which are governed by a transport equation. Simultaneously we obtain a weighted density function f by solving again the transport equation but with initial density as starting data. The averaged density is then resolved by the integration of the obtained f against the Dirac delta-function of two level set functions in the field space. Moreover, we prove that such obtained averaged density is simply a linear superposition of all multi-valued densities; and the averaged field quantities are weighted superposition of corresponding multi-valued ones. Computational results are presented and compared with some exact solutions which demonstrate the effectiveness of the proposed method.

More details on the derivation of level set method in capturing multi-valued velocity and electric fields and numerical examples can be found in [9]; details on computation and verification of superposition principle in density and other quantities can be found in [10].

2.2 Superposition of Multi-valued Solutions in High Frequency Wave Dynamics

The weakly coupled WKB system captures high frequency wave dynamics in many applications. For such a system a level set method framework has been recently developed to compute multi-valued solutions to the Hamilton–Jacobi equation and evaluate position density accordingly. In this project we propose two approaches for computing multi-valued quantities related to density, momentum as well as the energy. Within this level set framework we show that physical observables evaluated in [6, 7] are simply the superposition of their multi-valued correspondents. A series of numerical tests is performed to compute multi-valued quantities and validate the established superposition properties. More numerical details are in [11].

2.3 A Bloch Band Based Level Set Method for Computing the Semiclassical Limit in Schrödinger Equations

It is known that one can use level set description to accurately capture multi-phases in computation of high frequency waves. In this project, we develop a Bloch band based level set method for computing the semiclassical limit of Schrödinger equations in periodic media. For the underlying equation subject to a highly oscillatory initial data, a hybrid of the WKB approximation and homogenization leads to the Bloch eigenvalue problem and an associated Hamilton–Jacobi system for the phase, with Hamiltonian being the Bloch eigenvalues. We evolve a level set description to capture multi-valued solutions to the band WKB system, and then evaluate total position density over a sample set of bands. A superposition of band densities is established over all bands and solution branches when away from caustic points. The numerical approach splits the solution process into several parts: i) initialize the level set function from the band decomposition of the initial data; ii) solve the Bloch eigenvalue problem to compute Bloch waves; iii) evolve the band level set equation to compute multi-valued velocity and density on each Bloch band; iv) evaluate the total position density over a sample set of bands using Bloch waves and band densities obtained in step ii) and iii), respectively. Numerical results with different number of bands are provided to demonstrate the good quality of the method. Comparisons with direct simulation with small parameters are also provided. More details are in [12].

2.4 A Level Set Approach for Dilute Fluid-Particle Flows

Gas-particle and other dispersed-phase flows can be described by a kinetic equation containing terms for spatial transport, acceleration, and particle processes (such as evaporation or collisions). However, computing the dispersed velocity is a challenging task due to the large number of independent variables. A level set approach for computing dilute fluid-particle flows is presented. We will consider the sprays governed by the Williams kinetic equation with the initial distribution of the form $\sum_{i=1}^N \rho_i(x)\delta(\xi - u_i(x))$. The dispersed velocity is described as the zero level set of a smooth function, which satisfies a transport equation. This together with the density weight recovers the particle distribution at any time. Moments of any desired order can be evaluated by a quadrature formula involving the level set function and the density weight. It is shown that the method can successfully handle highly nonequilibrium flows (e.g. impinging particle jets, jet crossing, particle rebound off walls, finite Stokes number flows, etc.). More details are in [13].

2.5 Solutions to a Model Poisson-Nernst-Planck System and the Determination of Reaction Rates

The density distribution of molecular species is fundamental for understanding and describing biophysical and biochemical processes. The concentrations of ligands, receptors, and ions regulate nearly all

biomolecular and cellular activities. The Poisson–Nernst–Planck equation (PNP), is a proper physical model for describing the coupling of electrostatics, density distribution, and diffusion processes. Unlike the Smoluchowski–Nernst–Planck equation, in which the potential is considered as an external field, PNPE is a combination of Nernst–Planck (NP) equations and Poisson equation (PE), in which the potential is coupled. For nonsteady-state processes, or even steady-state processes with stationary values of diffusive fluxes, the PE must be used to determine the electrostatic field, because the nonequilibrium charge density distributions deviate from the Boltzmann distribution.

We study a modified PNP system for a charged spherical solute immersed in a solvent with multiple ionic or molecular species that are electrostatically neutralized in the far field. Some of these species are assumed to be in equilibrium. The concentrations of such species are described by the Boltzmann distributions that are further linearized. Others are assumed to be reactive, meaning that their concentrations vanish when in contact with the charged solute. We present both semi-analytical solutions and numerical iterative solutions to the underlying modified PNP system, and calculate the reaction rate for the reactive species. We give a rigorous analysis on the convergence of our simple iteration algorithm. Our numerical results show the strong dependence of the reaction rates of the reactive species on the magnitude of its far field concentration as well as on the ionic strength of all the chemical species. We also find non-monotonicity of electrostatic potential in certain parameter regimes. The results for the reactive system and those for the non-reactive system are compared to show the significant differences between the two cases. Our approach provides a means of solving a PNP system which in general does not have a close-form solution even with a special geometrical symmetry. Our findings can also be used to test other numerical methods in large-scale computational modeling of electro-diffusion in biological systems. More details are in the paper of [8].

2.6 Hydrophobic Interactions and Curvature Effect: A Level-set Variational Implicit-solvent Approach to a Concave Wall-ligand System

The correct description of solvation free energies and detailed solution structures of biomolecules is crucial to our understanding of molecular processes in biological systems. Efficient theoretical approaches to such descriptions are typically given by implicit (or continuum) solvent models of the aqueous environment. Although successful in many cases, the general applicability of many empirical implicit models with many system-dependent, adjustable parameters (e.g., individual atomic surface tensions) is often questionable, when compared to more accurate but computationally expensive explicit molecular dynamics (MD) simulations or experimental results. It is believed that the key issues here are the decoupling and separate analysis of surface area, dispersion and polar parts of the free energy, and the inaccurate free energy estimation due to a predefined solvent-solute interface, an *ad hoc* input. It is additionally well established by now that cavitation free energies do not scale with surface area for high curvatures, a fact of critical importance in the implicit modeling of hydrophobic interactions in biomolecular systems. Recently, Dzubiella, Swanson, and McCammon [4, 5] have developed a *variational* implicit solvent model, in which they introduced a free energy functional of all possible solute-solvent interfaces, coupling both the nonpolar and polar contributions of the system, and allowing for curvature correction of the surface tension to approximate the length-scale dependence of molecular hydration. Minimizing the functional leads to a partial differential equation whose solution determines the equilibrium solute-solvent interface and the minimum free energy of the solvated system.

A level-set method [1] is developed for numerically capturing the equilibrium solute-solvent interface that is defined by the recently proposed variational implicit solvent model. The evolution law is determined by minimization of a solvation free energy *functional* that couples both the interfacial energy and the van der Waals type solute-solvent interaction energy. The surface evolution is thus an energy minimizing process, and the equilibrium solute-solvent interface is an output of this process. We further explore this variational implicit model in a concave wall-ligand system [14] to capture the interfaces, fo-

cusing on the hydrophobic interactions and curvature effect. Simulation results show that this variational implicit model with level set method not only captures the hydrophobicity but also gives accurate energy by proper curvature correction. Some results are reported in [15] and more completed results are in [3].

2.7 The Level Set Method for Evolution of Constrained Hadwiger Surfaces with Application to Molecular Solvation

A level-set method is developed for the numerical minimization of a class of Hadwiger valuations with a potential on a set of three-dimensional bodies. Such valuations are linear combinations of the volume, surface area, and surface integral of mean curvature. The potential increases rapidly as the body shrinks beyond a critical size. The combination of the Hadwiger valuation and the potential is the mean-field free-energy functional of the solvation of non-polar molecules in the recently developed variational implicit-solvent model. This functional of surfaces is minimized by the level-set evolution in the steepest decent of the free energy. The normal velocity of this surface evolution consists of both the mean and Gaussian curvatures, and a lower-order, forcing term arising from the potential. The forward Euler method is used to discretize the time derivative with a dynamic time stepping that satisfies a CFL condition. The normal velocity is decomposed into two parts. The first part consists of both the mean and Gaussian curvature terms. It is of parabolic type with parameter correction, and is discretized by central differencing. The second part has all the lower-order terms. It is of hyperbolic type, and is discretized by an upwinding scheme. New techniques of local level-set method and numerical integration are developed. Numerical tests demonstrate a second-order convergence of the method. Examples of application to the modeling of molecular solvation are presented. More details in [2].

3 Current Research Projects

3.1 Development of Level Set Method in Variational Implicit Solvent Model with Coulomb and Yukawa Approximations

Central in the variational implicit-solvent model (VISM) [4, 5] is a mean-field free-energy functional that couples both the nonpolar and polar contributions of an underlying molecular system. Such a functional of all possible surfaces surrounding solute particles can be numerically relaxed by the powerful level-set method to determine the stable equilibrium solute-solvent interfaces and the solvation free energies. Initial tests have shown that the level-set variational implicit-solvent description of a nonpolar system is more accurate than that by a prefixed surface type implicit-solvent model, and can particularly capture the dewetting phenomenon in hydrophobic interaction. In this work, we treat the electrostatics in the VISM using the Coulomb-field and Yukawa-field approximations of electrostatic free energy without solving the Poisson or Poisson–Boltzmann equation. This approach is different from the generalized Born model in that no estimates of generalized Born radii are required. Instead volume integrals over arbitrary shaped regions need to be evaluated numerically. We also develop a fast level-set method with implicit time discretization to numerically minimize the free-energy functional for charged molecular systems. Applications to some model molecular systems show that our approach is accurate and efficient. In particular, our results agree qualitatively well with molecular dynamics simulations on the electrostatic effect to the hydrophobic interaction.

Numerical implementation and comparison with MD results have been done for simple molecular such as one atom, Benzene. We are testing more molecules such as two-nano-size-ball, BphC, etc.

3.2 Application and Analysis of Variational Implicit Solvent Model in Cylindrical Molecular

The variational implicit solvent model applies to all shapes of geometries. Spherical geometry has been extensively studied from many perspectives. Cylindrical shape, however, hasn't drawn much attention yet. The goal of this project is to theoretically and numerically analyze the stability of the system. We are particularly interested in the long time instability behavior introduced by the nonlinearity of the equation derived from the minimization of the energy. The cylindrical molecular is described by its radius, which we call a height function. By parametrizing in cylindrical coordinate and taking first variation of the corresponding energy functional, we reach a nonlinear PDE of the height function. Asymptotic analysis is then used to determine the long time instability. Simulation will be performed for numerical verification, too.

3.3 Level Set Method in Variational Implicit Solvent Model with Random Walk

The deterministic VISM method will not be able to capture all the physical states easily. We propose a stochastic approach to sample all the configuration states by slightly perturbing the position of the atoms. By the alternation of relaxation and perturbation, we are able to sample all the physical states. Due to the large computational cost in the Monte Carlo trials, the energy functional is simplified, i.e., we only consider volume, surface area and solute-solvent Leonard-Jones interaction. We are interested in testing the algorithm in two-atom system, two-plate system, pocket system, many-atom with harmonic potential system, etc. Some preliminary results have been obtained for the two-atom system and the results are comparable to the Molecular Dynamic results.

4 Research Plans

Besides the current projects, my future work includes:

1. Density evaluation in the semiclassical limit in Schrödinger equations
In our current level set method, the density is unbounded at the caustic point, which is not physically valid. It is natural to remove the unboundness in density by using other tools, such as Gaussian beam. Some effort has been made in this direction and more in-depth investigation is needed.
2. Recovering the original wave field ψ^ϵ using level set function(s)
The level set method based on WKB approximation gives the phase gradient, while the phase is needed in recovering ψ^ϵ . This requires integration of phase gradient and hence there is a constant yet to be determined. Our plan is to recover the phase and hence recover the wave field directly from the level set function(s).
3. Bohmian Mechanics in quantum dynamics
Recently there have been many interests in the Bohmian trajectories in the quantum dynamics. Similar to application of the level set method in the Schrödinger equation, I am interested in developing the level set method in Bohmian mechanics.
4. Application of the level-set method to VISM of electrostatics using Poisson-Boltzmann equation
In the polar molecules, the electric potential needs to be resolved separately in order to evolve the surface to an equilibrium state with locally lowest energy. The potential can be found by solving Poisson-Boltzmann equation, which gives the most accurate description of the electrostatics.
5. Geometric analysis in the level set method for variational implicit solvent model
In the level set equation of [1], the velocity (force) is given by a variation of an energy functional with respect to an interface. The energy functional depends on the surface geometry and hence

differential geometry is need for modeling the velocity (force). More analysis is need to improve the energy model.

6. Coupling fluid dynamics with particle crossing in two phase flow

In our previous work [12], in two phase flow, fluid is usually considered as external. Our next step is to couple the fluid and particle. This has been an very active research area in kinetic theory.

Level set method by nature has many advantages in handling movements, topological changes of any sharp interfaces and can be applied to many other areas. I am open to any new research projects or area in which level set method may be useful.

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