1. Please justify all answers and show your work.
2. You may work with classmates, but all answers must be your own.

1. Complete exercise 18 in section 1.1 and write your solutions here:

(a) \( f(4) = 2 \).

(b) Solutions to \( f(x) = 4 \): \( x = -3 \)

2. Suppose you are given the following piecewise function:

\[
 f(x) = \begin{cases} 
 x^3 + 1 & \text{if } x < 0 \\
 4 & \text{if } 0 \leq x \leq 3 \\
 3x + 1 & \text{if } x > 3 
\end{cases}
\]

(a) Evaluate \( f(-1) \). Show your work.

**Solution:** \( f(-1) = (-1)^3 + 1 = -1 + 1 = 0 \) since \(-1 < 0\).

(b) Evaluate \( f(2) \). Show your work.

**Solution:** \( f(2) = 4 \) since \( 0 \leq 2 \leq 3 \).

(c) Evaluate \( f(4) \). Show your work.

**Solution:** \( f(4) = 3 \cdot 4 + 1 = 13 \) since \( 4 > 3 \).
3. Determine the domains of the following functions. Write your answer in interval notation. Justify your answers.

(a) \( f(x) = 3 \cdot \sqrt{x - 2} \).

**Solution:** Since we can't take square roots of negative numbers, we need \( x - 2 \geq 0 \) or, equivalently, \( x \geq 2 \). Thus, the domain consists of all numbers greater than or equal to 2, which is \([2, \infty)\).

(b) \( g(x) = \frac{17}{x^2 - 4x + 3} \).

**Solution:** Since we can't divide by zero, we need to make sure we don't plug in values for \( x \) such that \( x^2 - 4x + 3 = 0 \). We can factor \( x^2 - 4x + 3 \) as \((x - 3)(x - 1)\). Thus, we need \( x \neq 3 \) and \( x \neq 1 \). Therefore the domain is \((-\infty, 1) \cup (1, 3) \cup (3, \infty)\).

(c) \( h(x) = 3 \cdot \sqrt{x - 2} + \frac{17}{x^2 - 4x + 3} \).

**Solution:** We can only plug in input values if we can plug them in to both \( f(x) = 3 \cdot \sqrt{x - 2} \) and \( g(x) = \frac{17}{x^2 - 4x + 3} \). Therefore we need \( x \geq 2 \), \( x \neq 3 \), and \( x \neq 1 \). Putting all this together, we see that our domain is \([2, 3) \cup (3, \infty)\).
4. Suppose your cell phone plan costs $60 and allows you to use up to 2 GB of data. You can also use additional data, but for all data that you use over 2 GB you are charged at a rate of $50 per GB.

(a) How much will you be charged if you use 2.5 GB of data? What about 5 GB of data?

Solution: If we use 2.5 GB of data, then we will have to pay $60 for our initial plan as well as

\[(0.5 \text{ GB}) \cdot (\$50/\text{GB}) = \$25\]

for the 0.5 GB we went over our 2 GB data allotment. Therefore will will be charged $85.

If we use 5 GB of data, then we will have to pay $60 for our initial plan as well as

\[3 \text{ GB} \cdot (\$50/\text{GB}) = \$150\]

for the 3 GB we went over our 2 GB data allotment. Therefore will will be charged $210.

(b) Write down a piecewise function $f$ which takes as input an amount of data (in GB) and outputs the amount (in dollars) that you are charged for using that much data.

Solution:

\[f(x) = \begin{cases} 
60 & \text{if } x \leq 2 \\
60 + 50 \cdot (x - 2) & \text{if } x > 2 
\end{cases}\]
(c) What are the domain and range of $f$?

**Solution:** Since we can (theoretically) use any nonnegative number of GB of data in a month, the domain of $f$ is $[0, \infty)$. Since we are charged a minimum of $60 and there is no limit to how much we can be charged, the range of $f$ is $[60, \infty)$.

(d) Sketch the graph of $f$.

**Solution:**

[Graph of the function $f$ showing a linear increase from $x=0$ to $x=6$, with y-axis values from 0 to 300.]
5. Exercise 1.3.8: Compute the average rate of change of the function $h(x) = 5 - 2x^2$ on the interval $[-2, 4]$. Show your work.

**Solution:** The average rate of change of $h(x)$ on $[-2, 4]$ is

$$\frac{h(4) - h(-2)}{4 - (-2)} = \frac{(5 - 2 \cdot 4^2) - (5 - 2 \cdot (-2)^2)}{6}$$

$$= \frac{(5 - 32) - (5 - 8)}{6}$$

$$= \frac{-27 - (-3)}{6}$$

$$= \frac{-24}{6}$$

$$= -4.$$
6. Consider the following function

\[
-8 - 6 - 4 - 2
\]

Estimate where this function

(a) is increasing:

**Solution:** \((-\infty, -2) \cup (0, \infty)\)

(b) is decreasing:

**Solution:** \((-2, 0)\)

(c) has local minima:

**Solution:** \(x = 0\)

(d) has local maxima:

**Solution:** \(x = -2\)

(e) is concave up:

**Solution:** \((-1, \infty)\)

(f) is concave down:

**Solution:** \((-\infty, -1)\).
7. The following table (data taken from Wikipedia) contains data on the number of confirmed COVID-19 cases in the U.S. For each input \( t \), \( C(t) \) is the number of confirmed COVID-19 cases in the U.S. up to the day which is \( t \) days after March 10th (I've arbitrarily chosen to start looking at the data at March 10). In other words, \( C(0) \) is the number of confirmed COVID-19 cases up to March 10, \( C(10) \) is the number of confirmed COVID-19 cases up to March 20th, etc.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( C(t) )</th>
<th>( t )</th>
<th>( C(t) )</th>
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<tbody>
<tr>
<td>0</td>
<td>936</td>
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<td>23,710</td>
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<td>1205</td>
<td>12</td>
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<td>1598</td>
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<td>15</td>
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<td>8074</td>
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<td>9</td>
<td>12,022</td>
<td>20</td>
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<tr>
<td>10</td>
<td>17,439</td>
<td>21</td>
<td>186,979</td>
</tr>
</tbody>
</table>

(a) Compute the average number of new COVID-19 cases per day between March 10th and March 20th. Show your work.

**Solution:** \( t = 0 \) corresponds to March 10 and \( t = 10 \) corresponds to March 20. Therefore the average number of new cases per day between March 10 and March 20 is

\[
\frac{C(10) - C(0)}{10 - 0} = \frac{17439 - 936}{10} = \frac{16503}{10}
\]

(b) Compute the average number of new COVID-19 cases per day between March 20th and March 31st. Show your work.

**Solution:** \( t = 10 \) corresponds to March 20 and \( t = 21 \) corresponds to March 31. Therefore the average number of new cases per day between March 20 and March 31 is

\[
\frac{C(21) - C(10)}{21 - 10} = \frac{186979 - 17439}{11} = \frac{169540}{11}
\]
(c) Here is a graph (from worldometers.info), which shows a graph of the above data (and more). Does this function appear to be concave up or concave down? How do the rates of change you computed previously support your conclusion?

Solution: The graph appears to be concave up. Our previous calculations support this conclusion because the average rate of change between March 10 and March 20 is less than the average rate of change between March 20 and March 31.