1. Find the equation of a line that passes through the points \((-2, 8)\) and \((4, 6)\). Show your work.

**Solution:** Our line has the form \(f(x) = mx + b\). We know that
\[
m = \frac{6-8}{4-(-2)} = -\frac{2}{6} = -\frac{1}{3}.
\]
We can plug in the point \((4, 6)\) to get
\[
6 = f(4) = \left(-\frac{1}{3}\right) \cdot 4 + b,
\]
which allows us to solve for \(b = 6 + \frac{4}{3} = \frac{22}{3}\). Hence, \(f(x) = -\frac{1}{3}x + \frac{22}{3}\).

2. Write an equation for a line perpendicular to \(f(x) = 3x + 4\) and passing through the point \((3, 1)\). Show your work.

**Solution:** Let’s denote our line by the \(g(x)\). We know \(g(x) = mx + b\) for some \(m\) and \(b\). Since \(g(x)\) is perpendicular to \(f(x)\), we know that \(m = -\frac{1}{3}\). We can then plug in the point \((3, 1)\):
\[
1 = g(3) = -\frac{1}{3} \cdot 3 + b
\]
and solve for \(b\) to get \(b = 2\). Hence, \(g(x) = -\frac{1}{3}x + 2\).
3. Suppose you get a new job as a salesperson. You are offered two different salary options: 
Option A: You are paid a base salary of $34,000 per year, plus a 12% commission on your 
sales (i.e. you get paid 12% of all that you sell) 
Option B: You are paid a base salary of $40,000 per year, plus a 5% commission on your 
sales (i.e. you get paid 5% of all that you sell) 

(a) Find a formula for a linear function \( f(x) \) such that 
\[
f(x) = \text{the amount you are paid for the year under option A if you make } x \text{ dollars in sales.}
\]
Do the same with a function \( g(x) \) for option B.

**Solution:**
\[
f(x) = 0.12x + 34000 \\
g(x) = 0.05x + 40000
\]

(b) How much would you need to sell for option A to produce a larger income? Show 
your work.

**Solution:** We need to compute the number of sales where option A and option 
B give the same income. In other words, we need to find the input \( x_0 \) such 
that \( f(x_0) = g(x_0) \). We can do this by setting \( f(x) \) and \( g(x) \) equal to each 
other and solving for \( x \):
\[
0.12x + 34000 = 0.05x + 40000 \\
.07x = 6000 \\
x \approx 85714.
\]
By sketching a graph of the two functions, we can see that \( f(x) > g(x) \) for 
all input values \( x \) after their intersection point (alternatively we can use test 
points). Therefore option A will give a greater income as long as our sales are 
greater than $85714.
4. Suppose we know that the median price of houses in California was $95,000 in 1980 and $450,000 in 2005. Assume that the median price of houses in California is a linear function of the year (in reality, this may not be a good assumption, but we will assume this for this problem). In other words, we are assuming that there is a linear function \( f(t) \) (which you will find in part (a)) that takes as input a number of years \( t \) and outputs the median price of houses in California \( t \) years after 1980.

(a) Define a function \( f(t) \) such that
\[
f(t) = \text{the median price of a house in California } t \text{ years after 1980}
\]

**Solution:** We are assuming that \( f(t) = mt + b \) for some \( m \) and \( b \). We know that \( f(0) = 95000 \) and \( f(25) = 450000 \). Therefore
\[
m = \frac{450000 - 95000}{25 - 0} = \frac{355000}{25} = 14200.
\]

We can then plug in one of our points to find \( b \):
\[
95000 = f(0) = 14200 \cdot 0 + b = b
\]
so that \( b = 95000 \). Thus, \( f(t) = 14200t + 95000 \).

(b) Using the function you defined above, when do we expect the median price of houses in California to be $1,000,000?

**Solution:** We expect the median price to be 1000000 in the year which is \( t_0 \) years after 1980, where \( f(t_0) = 1000000 \). We can figure out the value of \( t_0 \) by solving the following equation for \( t_0 \):
\[
1000000 = 14200 \cdot t_0 + 95000
\]
which gives \( t_0 \approx 63.7 \). This tells us that the median price will be $1000000 around 63.7 years after 1980, around the years 2043 to 2044.
5. Find an equation for the following line. Briefly justify your answer.

\[
\begin{align*}
5. \text{ Find an equation for the following line. Briefly justify your answer.}
\end{align*}
\]

\[
\begin{align*}
\text{Solution:} & \quad \text{We can see that the two points \((0,-1)\) and \((3,-3)\) are on the line. This gives us a slope of} \\
& \quad \quad \quad \quad m = \frac{-3 - (-1)}{3 - 0} = -\frac{2}{3}. \\
& \quad \text{Since the } y\text{-intercept is at } -1, \text{ this tells us that the equation of the line is } f(x) = -\frac{2}{3}x - 1.
\end{align*}
\]

6. Solve the inequality \(|2x - 9| + 1 < 8\). Show your work. (Note that the inequality here is strict (< as opposed to \(\leq\)) so if you follow the procedure from lecture, you'll need to make a very minor adjustment!)

\[
\begin{align*}
6. \text{ Solve the inequality } |2x - 9| + 1 < 8. \text{ Show your work. (Note that the inequality here is strict (< as opposed to } \leq) \text{ so if you follow the procedure from lecture, you'll need to make a very minor adjustment!)}
\end{align*}
\]

\[
\begin{align*}
\text{Solution:} & \quad \text{We first solve the equality } |2x - 9| + 1 = 8. \text{ Subtracting the 1 from both sides gives } |2x - 9| = 7. \text{ We know that } 2x - 9 = 7 \text{ when } x = 8 \text{ and } 2x - 9 = -7 \text{ when } x = 1. \text{ Thus, the solutions to } |2x - 9| + 1 = 8 \text{ are at } x = 1 \text{ and } x = 8. \text{ By looking at the graph of } |2x - 9| + 1, \text{ which looks similar to the usual absolute value function - a V-shape opening upward- we see that the inequality is satisfied for values in } (1,8). \quad (\text{Rather than looking at the graph, we could use test values instead}).
\end{align*}
\]
7. Let \( g(x) = -3x^5 + 2x^4 - 5x^2 + 8 \). Determine the behavior of \( g(x) \) as \( x \to \infty \) and as \( x \to -\infty \) (in other words, compute \( \lim_{x \to \infty} g(x) \) and \( \lim_{x \to -\infty} g(x) \)).

**Solution:** We know that the behavior of \( g(x) \) as \( x \to \pm \infty \) is the same as the behavior of its leading term, \(-3x^5\), as \( x \to \pm \infty \). Since \(-3x^5 \to -\infty \) as \( x \to \infty \) and \(-3x^5 \to \infty \) as \( x \to -\infty \), we see that

\[
\begin{align*}
  g(x) &\to -\infty \text{ as } x \to \infty \\
  g(x) &\to \infty \text{ as } x \to -\infty
\end{align*}
\]

as well.

8. Find the \( x \)-intercepts and \( y \)-intercepts of the function \( f(x) = 3(x + 1)(x - 4)(x + 5) \).

**Solution:** The \( y \)-intercept of \( f(x) \) is at

\[
f(0) = 3(0 + 1)(0 - 4)(0 + 5) = -60.
\]

\( f(x) \) has \( x \)-intercepts at input values \( x \) such that \( f(x) = 0 \) which is at \( x = -1, 4, -5 \).