Name: _____

PID: _____

1. Please justify all answers and show your work.
2. You may work with classmates, but all answers must be your own.

1. Make sure that when submitting your homework to Gradescope you indicate which page of your homework each question is on.

2. Find the slant asymptote of $f(x) = \frac{2x^3+x^2+x}{x^2+x+1}$. Show your work.

   We can perform polynomial division:

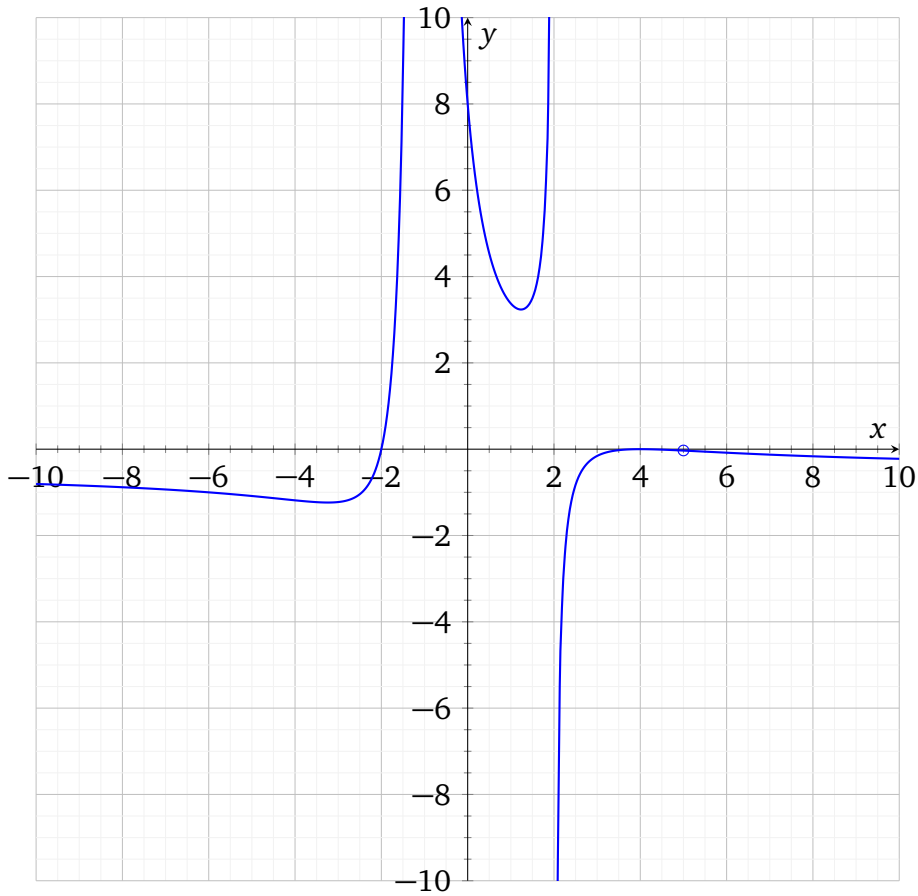
$$
\begin{array}{r}
2x-1 \\
x^2+x+1 \,\overline{)\; 2x^3+x^2+x} \\
\underline{-(2x^3+2x^2+2)} \\
-x^2-x \\
\underline{-(-x^2-x-1)} \\
1
\end{array}
$$

   This tells us that

$$f(x) = \frac{2x^3+x^2+x}{x^2+x+1} = 2x-1+\frac{1}{x^2+x+1},$$

   So the the slant asymptote is $2x-1$.

3. Find the holes, vertical asymptotes, horizontal asymptotes, x-intercepts, and y-intercepts of $f(x) = \frac{(x-5)(x+2)(x-4)^2}{-2(x-5)(x-2)(x+1)^2}$ (write them below the graph). Use this to sketch the graph. The sketch doesn't have to be exact, but make sure to use all the information you find above, and make sure your graph has the correct behavior at all the asymptotes and x-intercepts.



Solution: Hole where $x = 5$ since the term $(x-5)$ occurs in both the numerator and the denominator, and occurs to a power in the numerator which is greater than or equal to its power in the denominator.
Vertical asymptotes at $x = 2$ and $x = -1$ where the denominator is zero (but the numerator isn't)
Horizontal asymptote at $y = -1/2$, which is the ratio of the leading term of the numerator and the leading term of the denominator, since the numerator and denominator have the same degree.
x-intercepts at $(-2, 0)$ and $(4, 0)$ where the numerator is zero (and the denominator isn't).
y-intercept at $(0, f(0)) = (0, 8)$.

4. Find an equation for an exponential function which passes through the points (-3,4) and (2,3). Is this an exponential growth or decay function? Show your work.

Solution: We know that our exponential function has the form $f(x) = ab^x$ for some numbers a and b. Plugging in the first point gives us

$$4 = f(-3) = ab^{-3} = \frac{a}{b^3}.$$

Solving this equation for a tells us that $a = 4b^3$. Plugging in the second point gives us

$$\begin{aligned} 3 = f(2) = ab^2 &= (4b^3)b^2 && \text{since } a = 4b^3 \\ &= 4b^5. \end{aligned}$$

Solving this equation for b tells us that $b = (3/4)^{1/5}$. We can then plug this back into the equation $a = 4b^3$ to find a:

$$a = 4b^3 = 4\left((3/4)^{1/5}\right)^3 = 4(3/4)^{3/5}.$$

Hence, $f(x) = \left(4(3/4)^{3/5}\right)\left((3/4)^{1/5}\right)^x$.

5. Suppose you bought a car for \$28000 in 2010. In 2016, the value of the car was \$11000. Assuming the value of the car continues to decay at the same rate, find an equation for $f(t)$, where

$$f(t) = \text{ the car's value } t \text{ years after 2010.}$$

What is the value in 2020? Show your work.

Solution: The phrase "assuming the car continues to decay at the same rate" tells us that we are going to be modeling the value of the car with an exponential function. Hence, $f(t) = ab^t$ for some numbers a and b. We can use the same process as in problem 4. The fact that the value was \$28000 in 2010 tells us that

$$28000 = f(0) = ab^0 = a.$$

Then, the fact that the value was \$11000 in 2016 tells us that

$$11000 = f(6) = ab^6 = (28000)b^6.$$

Solving this equation for b gives us $b = (11000/28000)^{1/6}$. Hence,

$$f(t) = (28000)\left((11000/28000)^{1/6}\right)^t.$$

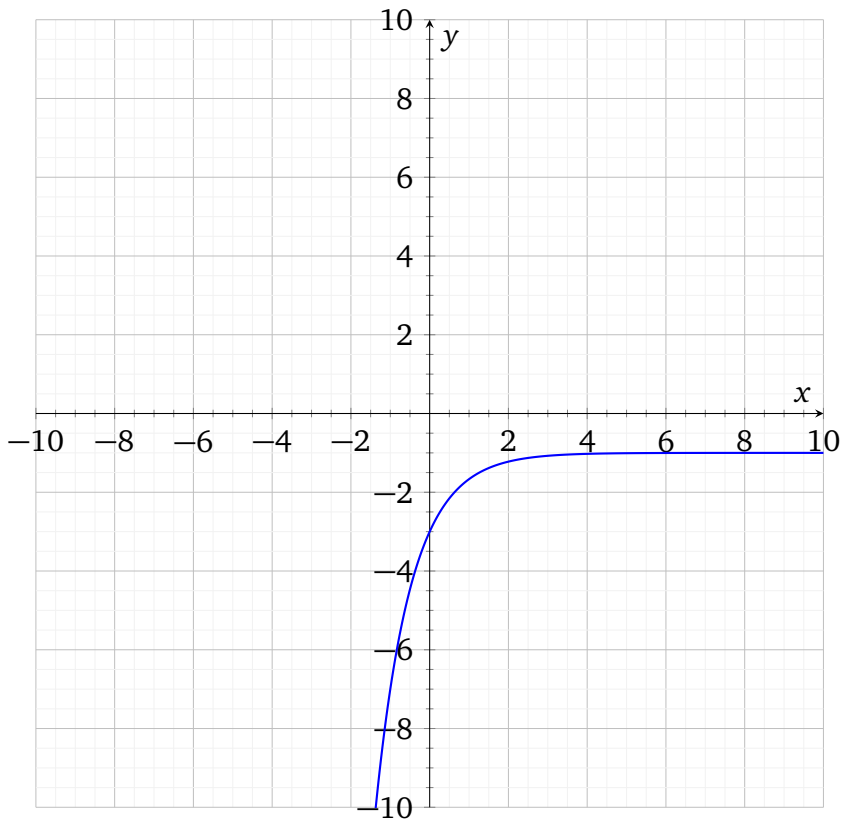
By plugging in $t = 10$ we get the value of the car in 2020 is $f(10) \approx 5900$ dollars.

6. Suppose a colony of bacteria starts with 1000 cells and the number of cells doubles every minute. Write a formula for a function which computes the number of bacteria cells in the colony after t minutes. Show your work.

Solution: Our funciton is of the form $f(t) = a(1+r)^t$ (I am using the $1+r$ form here since we are given a rate). Our initial value a is 1000, and our rate r is 1 (since doubling is the same as increasing by 100 %). Hence, $f(t) = 1000(2)^t$.

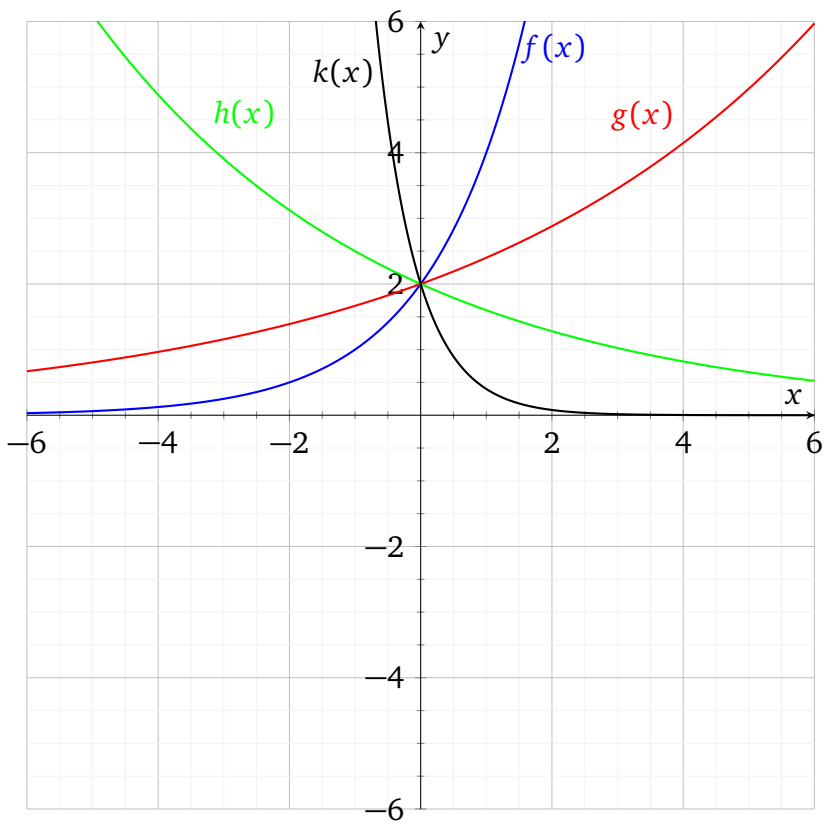
7. Describe the behavior of $f(x) = -2(3)^{-x} - 1$ as $x \to \infty$ and as $x \to -\infty$. Show your work.

Solution: We know that $f(x)$ can be obtained from the standard growing exponential function $g(x) = 2(3)^x$ by reflecting over both the x and y axes and then shifting down by 1. This tells us that $f(x) \to -1$ as $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$. It may help to sketch the graph using what we determined above:



8. Sketch graphs of the following functions on the same coordinate axes. Don't plot any points except for the y-intercepts; simply make sure they have the correct shape relative to each other (i.e. which functions are growing, which are decaying, which are growing faster than others, which functions are decaying faster than others?). Clearly label each function.

   1. $f(x) = 2(2)^x$
   2. $g(x) = 2(1.2)^x$
   3. $h(x) = 2(0.8)^x$
   4. $k(x) = 2(0.2)^x$



9. Let $f(x) = 3 \cdot 4^x$. Find equations for the following transformations of $f(x)$

   (a) $g(x)$, which is $f(x)$ shifted 3 units left.

   Solution: $g(x) = f(x+3) = 3 \cdot 4^{x+3}$

   (b) $h(x)$, which is $f(x)$ reflected over the y-axis.

   Solution: $h(x) = f(-x) = 3 \cdot 4^{-x}$.

   (c) $k(x)$, which is $f(x)$ vertically stretched by a factor of 8.

   Solution: $k(x) = 8 \cdot f(x) = 24 \cdot 4^x$.