Name: _____

PID: _____

1. Please justify all answers and show your work.
2. You may work with classmates, but all answers must be your own.

1. Make sure that when submitting your homework to Gradescope you indicate which page of your homework each question is on.
2. In each problem below, use properties of logarithms and exponentials to simplify each expression without a calculator. Show your work.
   (a) $\ln\left(e^{-2}\right)$

   (b) $y^{2\log_y(4)}$

3. Solve the equation $\log_4(x) = 3$ without using a calculator. Show your work.

4. Solve the following equations. You may leave your answer in terms of logarithms or you can plug your answer into a calculator to get a decimal approximation. Show all of your work.

   (a) $200(1.06)^t = 550$

   (b) $100 - 100\left(\frac{1}{4}\right)^x = 70$

   (c) $2^{3x-2} = 5^{4x-3}$

5. Solve the following equations. Show all of your work.

   (a) $\log_2(-x) - \log_2(-x^3) = 4$.

   (b) $\ln(x) + \ln(x-6) = \ln(6x)$

   (c) $\log_{10}(x+4) + \log_{10}(x) = 9$

6. A scientist starts with 100 mg of a radioactive substance which decays exponentially. After 6 days, it has decayed to 60 mg. What is the half-life of the radioactive substance? How long will it take to decay to 10 mg (from the original 100 mg)?

7. You are an unethical zoo owner looking to make some money by breeding your tigers and selling them. You currently have 2 tigers. Assume that every pair of tigers you have can produce another tiger every 4 months. For example, 4 months after starting you will have $2 + 2(0.5) = 2(1+0.5) = 3$ tigers. Ignore any practical concerns about what it would mean to have a fraction of a tiger.

   (a) Above, you are told that your tiger population increases by 50% every four months. Compute the rate at which your tiger population is increasing every year. Use this to define an exponential growth function $f(t)$ where

   $$f(t) = \text{your tiger population } t \text{ years after you start your breeding program.}$$

   (b) Suppose another zoo wants to buy 10000 tigers from you. Calculate how many years it will take you to go from the 2 tigers you start with to 1000 tigers.

(c) Suppose that after 3 years of breeding your tigers, your rival tiger zoo owner notices the success of your breeding program and starts to breed their own tigers. At that point, you have $f(3)$ amount of tigers, and your rival has 2 tigers. However, your rival is even more unethical than you are, and has found a way to have their tigers reproduce every 3 months instead of every four months. You and your rival are competing to have the largest tiger population. Write a function $g(t)$ that computes your rival's tiger population t years after they start their breeding program. Use this to compute how many years it takes for your rival's tiger population to surpass yours. (Hint: you may want to rewrite the function that computes your tiger population to be a new function $h(t)$ which computes your tiger population t years *after your rival starts their breeding program* - a horizontal shift of your original function $f(t)$).