Name: _____

PID: _____

1. Please justify all answers and show your work.
2. You may work with classmates, but all answers must be your own.

1. Make sure that when submitting your homework to Gradescope you indicate which page of your homework each question is on.

2. (a) Compute the reference angle for the angle $\frac{7\pi}{6}$ and use it (along with your knowledge of the values of $\sin(\theta)$ and $\cos(\theta)$ for common angles in the first quadrant) to compute $\sin\left(\frac{7\pi}{6}\right)$. Show your work.

Solution: Since $\frac{7\pi}{6}$ is in the third quadrant (an easy way to see this is the fact that $1 \leq \frac{7}{6} \leq \frac{3}{2}$ implies that $\pi \leq \frac{7\pi}{6} \leq \frac{3\pi}{2}$ (multiplying the two inequalities by π), its reference angle is

$$\frac{7\pi}{6} - \pi = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}.$$

This tells us that either $\sin\left(\frac{7\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$ or $\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$. Since θ is in the third quadrant, where sine is negative, we conclude that

$$\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}.$$

(b) Compute the reference angle for the angle $\frac{2\pi}{3}$ and use it (along with your knowledge of the values of $\sin(\theta)$ and $\cos(\theta)$ for common angles in the first quadrant) to compute $\cos\left(\frac{2\pi}{3}\right)$. Show your work.

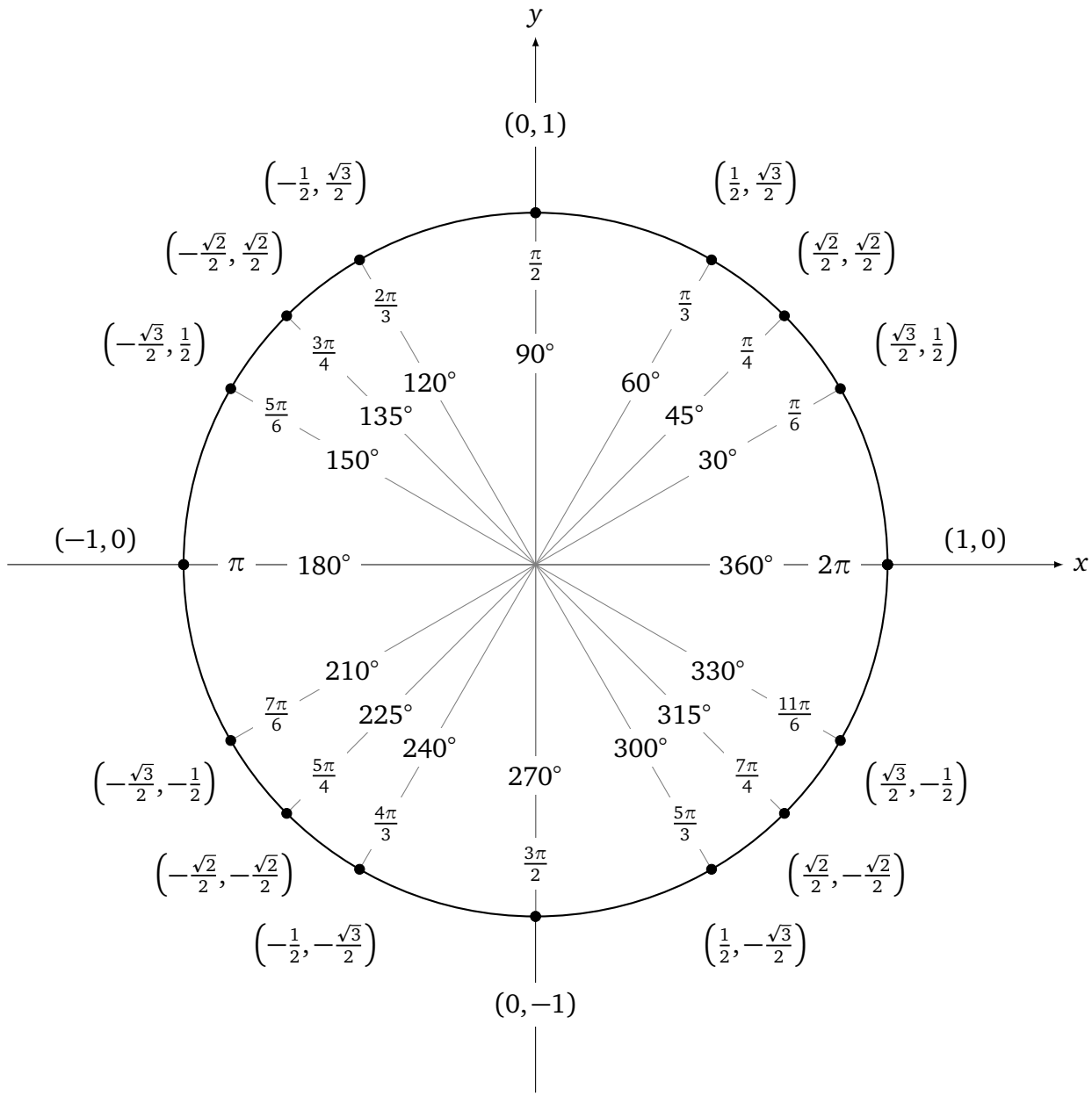
Solution: Since $\frac{2\pi}{3}$ is in the second quadrant, its reference angle is

$$\pi - \frac{2\pi}{3} = \frac{3\pi}{3} - \frac{2\pi}{3} = \frac{\pi}{3}.$$

This tells us that either $\cos\left(\frac{2\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right)$ or $\cos\left(\frac{\pi}{3}\right) = -\cos\left(\frac{2\pi}{3}\right)$. Since θ is in the second quadrant, where cosine is negative, we conclude that

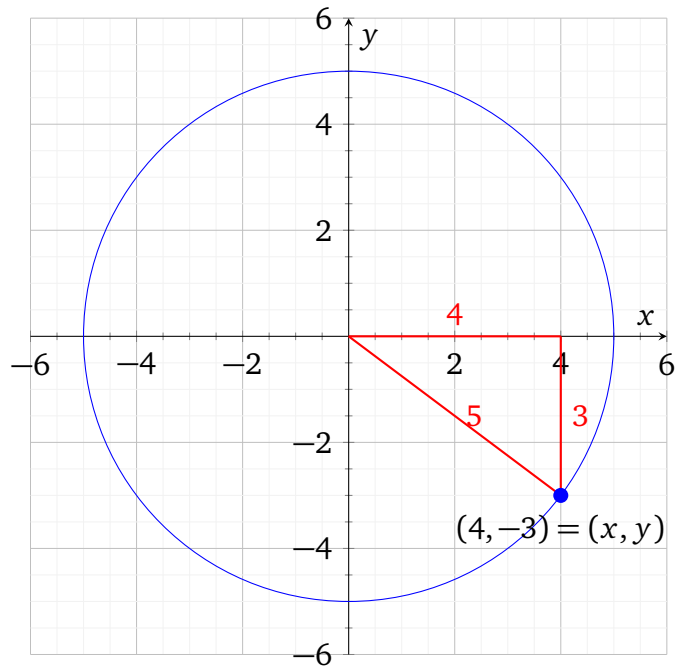
$$\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}.$$

3. Fill in the points at each of the following angles on the unit circle ("unit circle" means it has radius 1). Remember that $x = r\cos(\theta) = \cos(\theta)$ (since $r = 1$ here) and $y = r\sin(\theta) = \sin(\theta)$ (since $r = 1$ here), so you'll need to calculate the values of $\sin(\theta)$ and $\cos(\theta)$ for these angles.



4. Suppose θ is an an angle such that $\sin(\theta) = -\frac{3}{5}$, and θ is in the fourth quadrant. What are $\cos(\theta), \tan(\theta), \csc(\theta), \sec(\theta)$, and $\cot(\theta)$? Show your work.

Solution: The following picture describes our situation:



We can view our angle θ as being in a circle of radius 5, in which case its terminal edge is the diagonal red line (labeled "5") above. The fact that $\sin(\theta) = -\frac{3}{5}$ tells us that the y-coordinate of the corresponding point on the circle is -3 (since we have chosen the circle to have radius 5 and we need $\sin(\theta) = \frac{y}{r} = -\frac{3}{5}$). We can then use the Pythagorean Theorem to compute that the other side of the red triangle must have length 4. Since we are in the fourth quadrant, this tells us that the x-coordinate of our point is 4. We can then use the definitions of the other trigonometric functions to compute them:

$$\cos(\theta) = \frac{4}{5}$$
$$\tan(\theta) = \frac{-3}{4}$$
$$\csc(\theta) = \frac{-5}{3}$$
$$\sec(\theta) = \frac{5}{4}$$
$$\cot(\theta) = \frac{4}{-3}$$

5. Simplify the following trig expressions to an expression involving a single trig function with no fractions (for example, something of the form $\sec(x)$ or $\cot^2(x)$, etc.). Problems like these are not always straightforward, but you can try the methods we used in lecture - writing everything in terms of sin and cos, multiplying by $\frac{\sin(x)}{\sin(x)}$ (or something similar), and using the Pythagorean Identity.

   (a) $\frac{\cot(x)}{\csc(x)} \cdot \sec(x)$.

   Solution:

$$\begin{aligned}\frac{\cot(x)}{\csc(x)} \cdot \sec(x) &= \frac{(\cos(x)/\sin(x))}{1/\sin(x)} \cdot \frac{1}{\cos(x)} && \text{writing cot, csc, sec in terms of sin and cos}\\ &= \frac{\cos(x)}{\sin(x)} \cdot \frac{\sin(x)}{1} \cdot \frac{1}{\cos(x)}\\ &= 1.\end{aligned}$$

   (b) $\frac{1-\sin^2(\theta)}{\sin^2(\theta)}$.

   Solution:

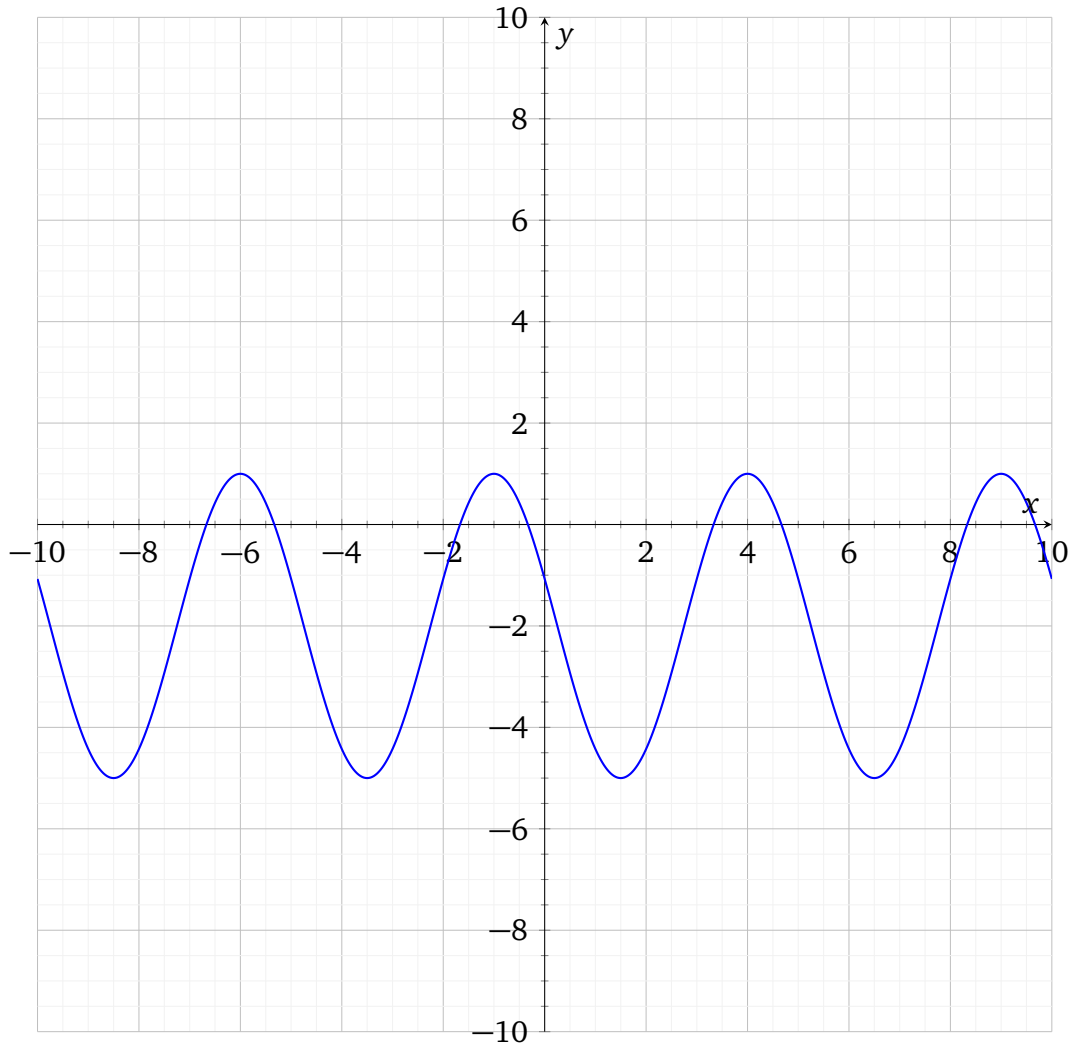
$$\begin{aligned}\frac{1-\sin^2(\theta)}{\sin^2(\theta)} &= \frac{\cos^2(\theta)}{\sin^2(\theta)} && \text{by the Pythagorean Identity}\\ &= \cot^2(\theta) && \text{since } \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}\end{aligned}$$

   (c) $\frac{\sec(t)-\cos(t)}{\sin(t)}$.

   Solution:

$$\begin{aligned}\frac{\sec(t)-\cos(t)}{\sin(t)} = \frac{(1/\cos(t))-\cos(t)}{\sin(t)} &= \frac{(1/\cos(t))-\cos(t)}{\sin(t)} \cdot \frac{\cos(t)}{\cos(t)}\\ &= \frac{1-\cos^2(t)}{\sin(t)\cos(t)}\\ &= \frac{\sin^2(t)}{\sin(t)\cos(t)}\\ &= \frac{\sin(t)}{\cos(t)}\\ &= \tan(t).\end{aligned}$$

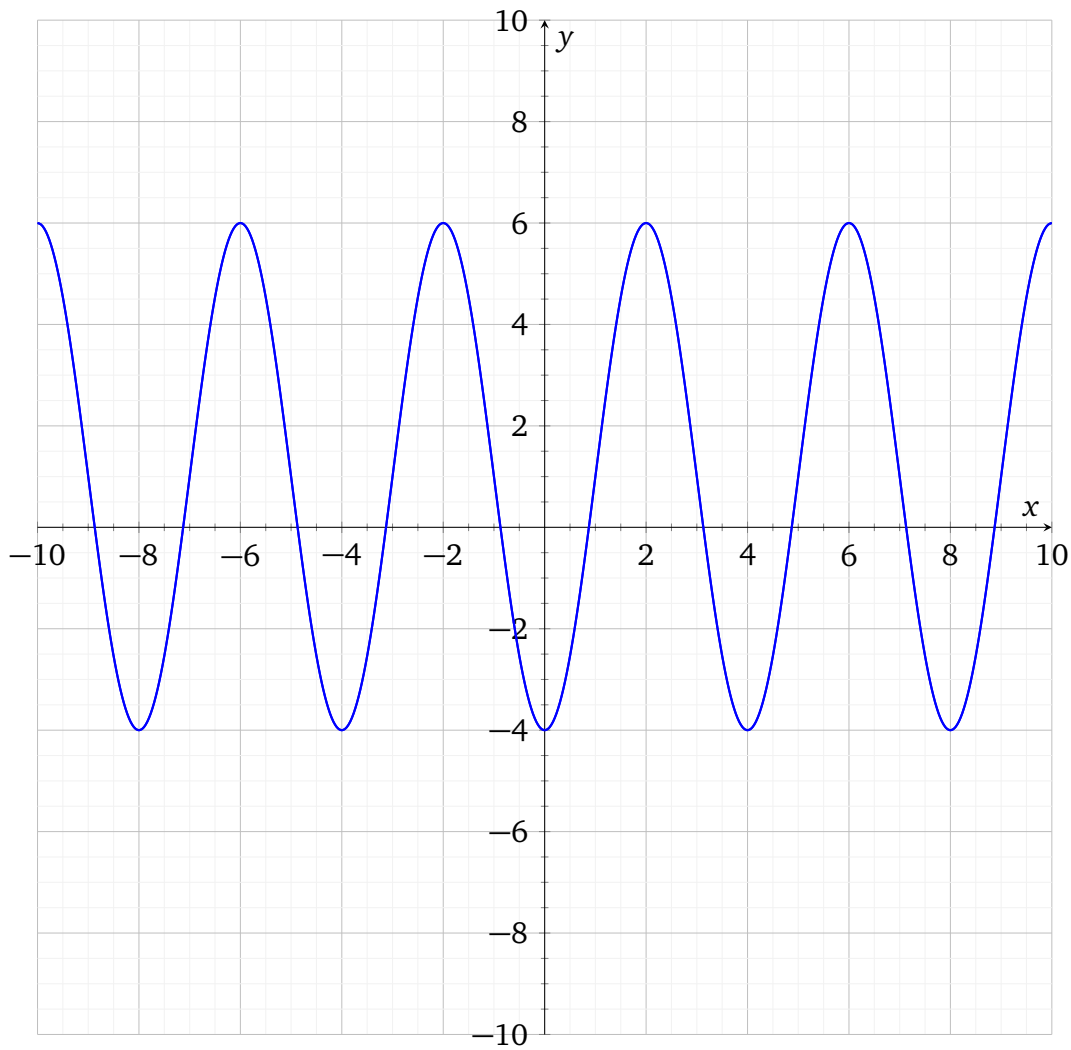
6. What are the amplitude, period, midline, and horizontal shift of the sinusoidal function $f(t) = 3\cos\left(\frac{2\pi}{5}(x+1)\right) - 2$? Write this information below the axes and use it to sketch the graph of $f(t)$. Label the amplitude, period, and midline on the graph.



The amplitude is 3, the period is $\frac{2\pi}{2\pi/5} = 5$, the midline is $y = -2$, and the horizontal shift is 1 to the left.

7. (a) Write an equation for the function graphed below which is in the form $f(x) = A\sin(B(x-h)) + k$ (you need to figure out what A, B, h, and k are).

   (b) Write an equation for the function graphed below which is in the form $g(x) = A'\cos(B'(x-h')) + k'$ (you need to figure out what A', B', h', and k' are).



Solution: We will do part (b) first. We know that our midline is halfway between the maximum and minimum values attained by our sinusoidal function, so the midline is $y = 1$. The amplitude is the difference between our maximum value attained by our function and our midline, so the amplitude is 5. The period is the horizontal distance between two adjacent local maxima on the graph, so the period is 4. Finally, since cosine normally has a local max at $x = 0$ and our graph has a local max at $x = 2$, we can use a horizontal shift 2 to the right (of course, due to periodicity, horizontal shifts of 2,6,10,14, etc. to either the right or left work). Therefore our function is

$$g(x) = 5\cos\left(\frac{2\pi}{4}(x-2)\right) + 1.$$

To write this in terms of sin, the amplitude, midline, and period are the same, but the horizontal shift changes. We can find the horizontal shift from the graph, or we can use the fact that $\cos(\theta) = \sin\left(\theta + \frac{\pi}{2}\right)$ to see that

$$\begin{aligned}
5\cos\left(\frac{2\pi}{4}(x-2)\right) + 1 &= 5\sin\left(\frac{2\pi}{4}(x-2) + \frac{\pi}{2}\right) + 1 \\
&= 5\sin\left(\frac{2\pi}{4}(x-2+1)\right) + 1 \quad \text{factor } \frac{\pi}{2} \text{ out from both terms} \\
&= 5\sin\left(\frac{2\pi}{4}(x-1)\right) + 1
\end{aligned}$$