1. Make sure you can solve all homework problems.

2. How can we tell if a graph is the graph of a function or not?

3. How can we tell if a relationship defined by a table or by an equation is a function or not?

4. What does it mean for a function to be one-to-one? How can we tell if a function is one-to-one from it's graph?

   (a) Which of the following functions are one-to-one? How do you know?
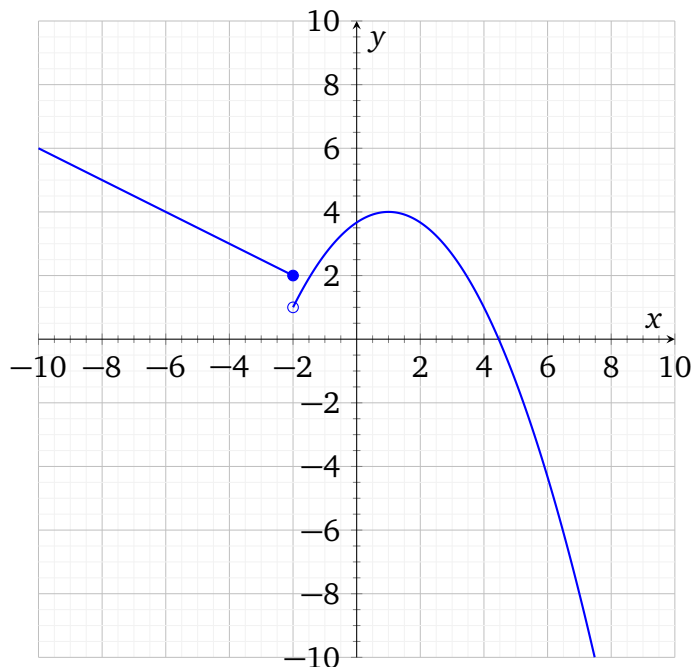
$$f(x) = x$$
$$g(x) = x^2$$
$$h(x) = x^3$$

5. What is the domain of a function? What is the range of a function?

   (a) What are the domain and range of $f(x) = -2 \cdot \frac{x+2}{x-4}$?

   (b) What are the domain and range of the function $g(x)$ defined by the table

x	3	5	8	10
g(x)	1	-9	3	3

6. Draw the graph of the piecewise function

$$f(x) = \begin{cases} 3x + 2 & \text{if } x < -2 \\ (1/2)x - 1 & \text{if } x \geq 3 \end{cases}$$

7. Find the equation of the following piecewise function

8. Given a function $f(x)$, how do we compute the average rate of change of $f(x)$ on the interval $[a, b]$?

   (a) Compute the average rate of change of $f(x) = 6x^2 + \frac{4}{x^3}$ on $[-1, 3]$.

   (b) Compute the average rate of change of $h(x) = \frac{1}{x+3}$ on $[1, 1+a]$.

9. What does it mean for a function to be increasing or decreasing? What is a local minimum/maximum? What does it mean for a function to be concave up or concave down? What does it look like when $f(x) \to \infty$ as $x \to \infty$? What about $f(x) \to -\infty$ as $x \to \infty$? What about $f(x) \to c$ as $x \to \infty$ for some number c?s

   (a) Estimate where the following function $g(x)$ is increasing/decreasing and concave up/concave down? What is the behavior of $g(x)$ as $x \to \infty$ and as $x \to -\infty$.
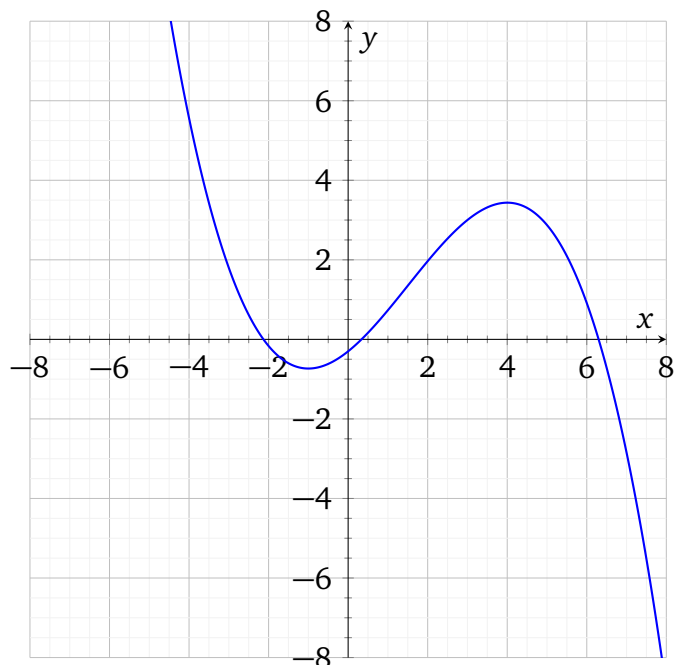
(b) Estimate where the following function $f(x)$ is increasing/decreasing and concave up/concave down? What is the behavior of $f(x)$ as $x \to \infty$ and as $x \to -\infty$.
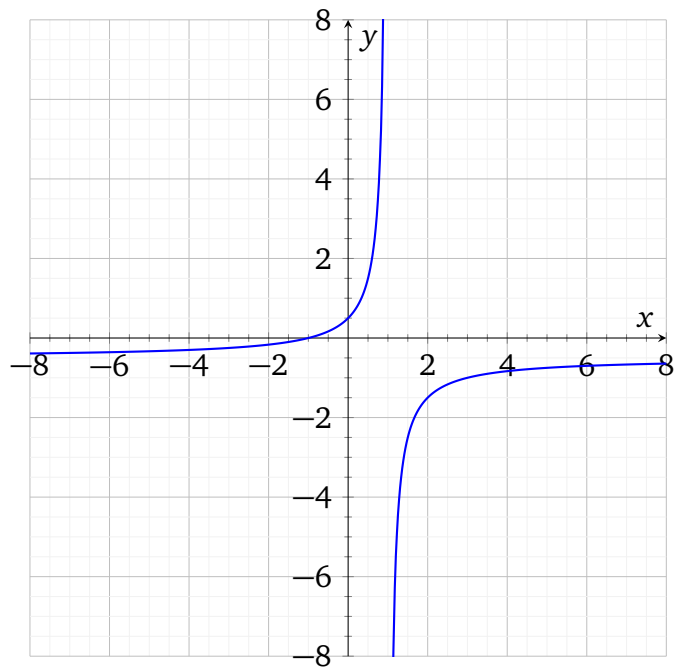

10. Compute the following compositions.

(a) $f(x) = \sqrt{x} + 2$, $g(x) = x^2 + 3$, compute $f(g(x))$ and $g(f(x))$ and find their domains.

(b) $f(x) = \frac{1}{x-3}$, $g(x) = \frac{1}{x} + 3$, compute $f(g(x))$ and $g(f(x))$ and find their domains.

11. Find functions $f(x)$ and $g(x)$ such that $h(x) = \frac{4}{(x+2)^2}$ is equal to $h(x) = f(g(x))$.

12. Make sure you know the following transformations of functions - how they affect the graph of the function and how they appear in the equation:

   1. vertical shift

   2. horizontal shift

   3. vertical reflection

   4. horizontal reflection

   5. vertical stretch/compression

   6. horizontal stretch/compression

13. Make sure you know how to combine multiple transformations - both in the equation for the function and also in the graph of the function.

14. Make sure you know how each of the transformations affects vertical asymptotes, horizontal asymptotes, x and y-intercepts, domains and ranges.

15. Find a formula for the following function, which is a transformation of $\sqrt{x}$
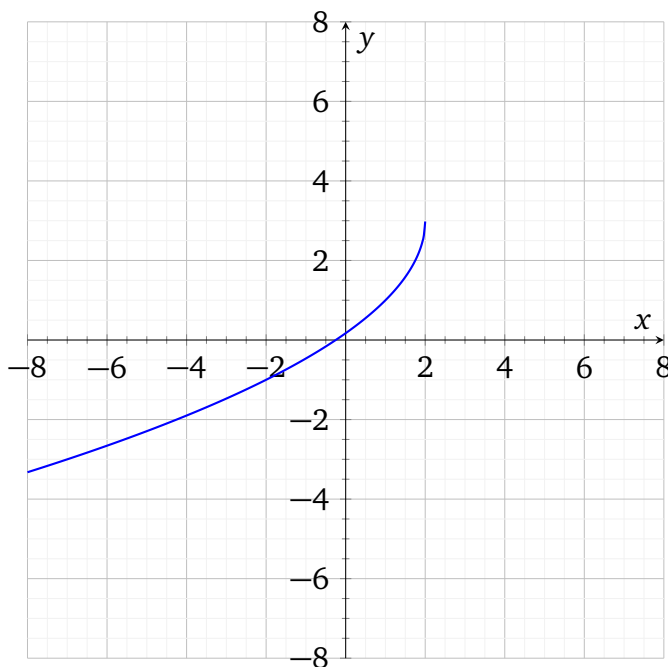
16. Let $f(x) = |x|$. Write an equation for the transformation of $f(x)$ which consists of a horizontal shift right 2, vertical shift down 1, vertical reflection, and vertical stretch by a factor of 3. Sketch the graph of this transformation.

17. When does a function $f(x)$ have an inverse?

    (a) Does $f(x) = 6x - 2$ have an inverse? If so, what is it?

    (b) Does $g(x) = 6x^2 - 2$ have an inverse? If so, what is it?

    (c) Does $h(x) = 6x^3 - 2$ have an inverse? If so, what is it?

18. What does the graph of $f^{-1}(x)$ look like (compared to the graph of $f(x)$)?

19. What are the domain and range of $f^{-1}(x)$ (compared to the domain and range of $f(x)$)?

20. Assuming 2 is in the domain of $f(x)$, what is $f^{-1}(f(2))$?

21. If $f(3) = 6$ and $f(6) = 9$, what is $f^{-1}(6)$?

22. What is the inverse of $h(x) = \frac{x}{2+x}$?

23. Write the equation of a line that passes through the points (-6,-2) and (3,7). What is the slope? What is the y-intercept?

24. Write the equations of the lines graphed here
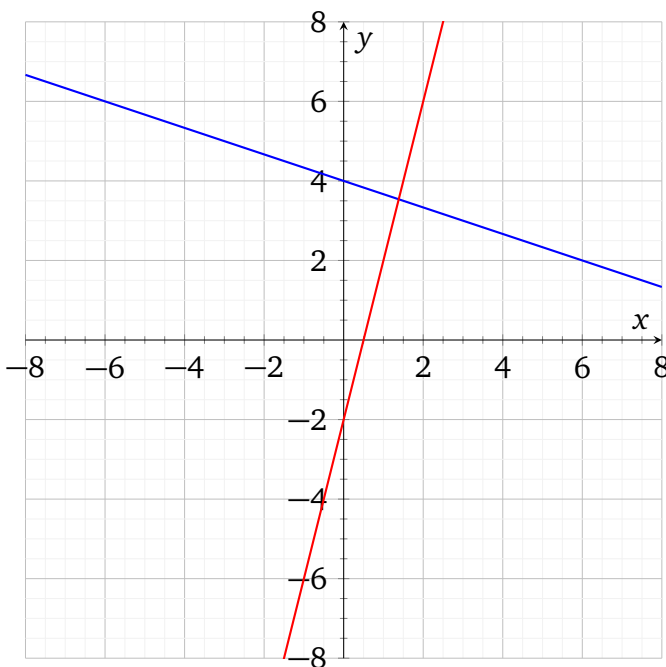
25. Graph the function $f(x) = \frac{2}{5}x + 1$

26. Assuming m and b are numbers with $m \neq 0$, what is the x-intercept of $f(x) = mx + b$?

27. Write the equation of a line which is parallel to $g(x) = \frac{1}{3}x - 2020$ and passes through (2,2).

28. Write the equation of a line which is perpendicular to $g(x) = \frac{1}{3}x - 2020$ and passes through (-1,3).

29. Do the 2 previous lines you found intersect? If so, at which point? If not, then why not?

30. Solve the absolute value inequality $-3|2(x+1)| + 6 \leq -2$. Sketch the graph of $f(x) = -3|2(x+1)| + 6$.

31. Determine the behavior of the polynomial $g(x) = -6x^9 + 5x^6 - x^5 + x + 10$ as $x \to \infty$ and as $x \to -\infty$.

32. Find an equation of a quadratic function which has vertex $(-4, 6)$ and which contains the point (1,-1).

33. Find an equation of a quadratic function which has x-intercepts $(5, 0)$ and $(-3, 0)$ and which contains the point (1,-4).

34. Find the roots (x-intercepts) and the vertex of the quadratic polynomial $f(x) = 6x^2 - 14x + 3$.

35. A farmer wants to enclose 4 pens as shown:
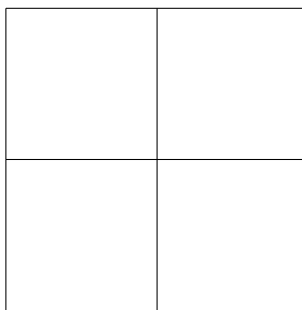
The farmer has 600 feet of fence to work with. What is the maximum area the farmer can enclose?

36. A ball is thrown in the air. Its height (in meters) t second after being thrown is given by $h(t) = -4.9t^2 + 12t + 3$. What height is the ball thrown from? What is the highest point the ball reaches and at what time does it reach it? When does the ball hit the ground?

37. Find the domain of $k(x) = \sqrt{(x-1)^2(x+3)}$.

38. Find the domain of $h(x) = \frac{3x^8 - 4x + 1}{x^2 - 4x - 5}$

39. Find a degree 4 polynomial with a root of multiplicity 2 at $x = 4$, and roots of multiplicty 1 at $x = 1$ and $x = -2$, as well as a y-intercept at $(0, -3)$.

40. Find the horizontal asymptote of each function or, if it does not have one, find the slant asymptote.

   (a) $f(x) = \frac{2x^2 + 7x - 15}{3x^2 - 14x + 15}$

   (b) $g(x) = \frac{2x^2 + 7x - 15}{x - 5}$

   (c) $h(x) = \frac{x - 5}{2x^2 + 7x - 15}$

41. Sketch the graph of $f(x) = \frac{2(x-1)(x+2)^2(x-5)}{(x-1)(x-3)^2(x+4)}$

42. The following is the graph of a function of the form $f(x) = m\frac{(x-a)(x-b)}{(x-c)(x-d)}$ for some numbers m, a, b, c, d (that may be equal). Determine what m, a, b, c, d are.
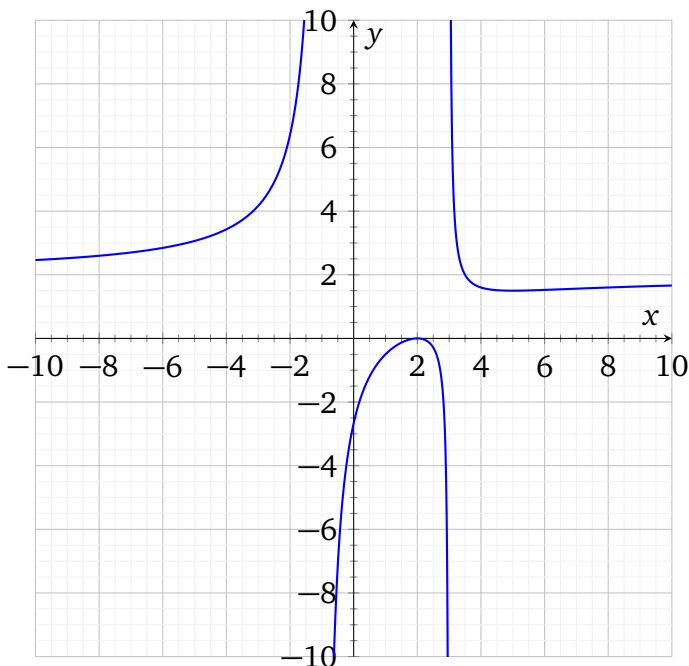
43. Write an exponential function that models a population which starts at 1000 and grows by 6% per year.

44. Write an exponential funciton that models the value of an item which starts at $2000 and decays by 12% each year.

45. Find an exponential function which passes through the points (2,5) and (6,9).

46. You start with 110 mg of a radioactive substance. After 31 hours, 55 mg remain. Find an exponential function which models the decay of the substance. How many milligrams will remain after 42 hours?

47. There were 1200 deer that live in Forest Park in 2015. There are 1700 deer there now (2020). Write an exponential function that models the population of deer in Forest Park. Predict how many deer there will be there in 2025.

48. Sketch a graph of the transformed exponential function $f(x) = 3(1/2)^x - 2$.

49. Find a formula for the transformed exponential function graphed below.
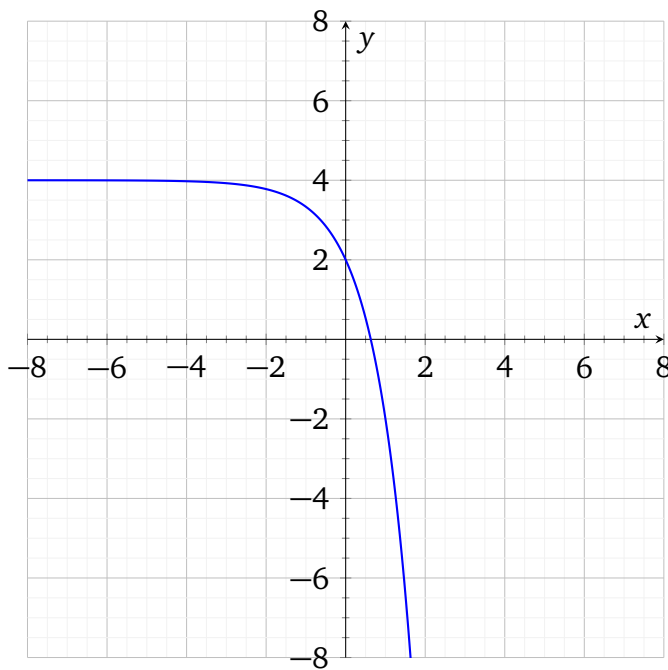
50. Match each of the following functions with its graph

      1. $f(x) = 2(1.3)^x$

      2. $g(x) = 2(1.8)^x$

      3. $h(x) = 4(1.3)^x$

      4. $k(x) = 4(0.7)^x$.