Problem 1 Draw the graph of the following functions. Details don’t matter, but you must indicate the correct horizontal and vertical asymptotes, x- and y-intercepts, whether it’s increasing or decreasing, concave up or down.

a) (5 pts) \( f(x) = \frac{2x+1}{3x-1} \)

b) (5 pts) \( g(x) = \frac{-2x-3}{3x-1} \)
Problem 2

a) (5 points) Find the domain, then draw the graph of the function \( f(x) = \frac{x^2 - 3x + 2}{x^2 - 4x + 3} \), the requirements are the same as in the last problem. (Hint: The numerator may share a common factor with the denominator.)

b) (5 points) Find the domain and the slant asymptote of the function \( f(x) = \frac{2x^3 + 4x^2 - 2x + 1}{3x^2 + 2} \)
Problem 3 (10 points) Does the function $f(x) = x^2 - 6x + 6$ have an inverse function? Explain why. If it does, find the inverse function $f^{-1}$, otherwise, find the smallest number $a$ such that $f(x)$ has an inverse function on $[a, \infty)$, and find $f^{-1}$ on this given interval.
Problem 4

a) (3 points) Suppose there are a million bacteria in a biology lab now, after every minute, each of them will split into two new ones. For example, after 1 minute, there are 2 million in the lab. How many bacteria will there be after \( n \) minutes? Write your answer as a function of \( n \).

b) (3 points) If you are given \( n \) multiple choice problems, each problem has 4 choices. How many different answers can there possibly be? Briefly explain why it’s impossible to get good grade without study for such an exam that has many multiple choice problems.

c) (4 points) A radioactive substance decays exponentially. A scientist begins with 100 milligrams of a radioactive substance. After 35 hours, 50 mg of the substance remains. How many milligrams will remain after 54 hours? What is the half life of the substance?
Problem 5

a) (5 points) Find an exponential function that passes through the two points (4, 12) and (6, 24).

b) (5 points) Is the function you find in the last part increasing or decreasing? Does it have doubling time or half life? Find the doubling time or half life if it has one.
Problem 6 (10 points) The MIT museum contains a kinetic sculpture by Arthur Ganson called “Machine with concrete” (watch this video: https://www.youtube.com/watch?v=5q-BH-tvxEg). It consists of 13 gears connected to one another in a series such that each gear moves 50 times slower than the previous one. The fastest gear is constantly rotated by an engine at a rate of 212 rotations per minute. The slowest gear is fixed to a block of concrete and so apparently cannot move at all. How come this machine does not break apart? Justify your answer with concrete computations.