For the other practice problems, see the study guides for the two midterms.

Problem 11:

(a) \( \overrightarrow{AB} = (\cos t, \sin t) \) and \( \overrightarrow{OA} = (10t, 0) \), so \( \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = (10t + \cos t, \sin t) \).

The rear bumper is reached at time \( t = \pi \) and the position of \( B \) is \((10\pi - 1, 0)\).

(b) \( \vec{v}(t) = (10 - \sin t, \cos t) \), so

\[
|\vec{v}|^2 = (10 - \sin t)^2 + \cos^2 t = 100 - 20\sin t + \sin^2 t + \cos^2 t = 101 - 20\sin t.
\]

The speed is then given by \( |\vec{v}| = \sqrt{101 - 20\sin t} \). 

The speed is smallest when \( \sin t \) is largest i.e. \( \sin t = 1 \), which occurs when \( t = \pi/2 \). At this time, the position of the bug is \((5\pi, 1)\).

The speed is largest when \( \sin t \) is smallest; that happens at the times \( t = 0 \) or \( \pi \) for which the position is then \((0, 0)\) and \((10\pi - 1, 0)\).

Problem 28:

\[
\text{Mass}(R) = \iiint_R \rho(x, y, z)dV = \iiint_R ydV.
\]

The equation of the sphere is \( x^2 + y^2 + (z - 2)^2 = 16 \).

The shadow of \( R \) on the \( xy \)-plane is determined by the intersection of these two paraboloids. In other words, we need \( z = 4 - x^2 - y^2 \) to sit underneath \( z = 10 - 4x^2 - 4y^2 \), i.e. \( 4 - x^2 - y^2 \leq 10 - 4x^2 - 4y^2 \). That is, we need \( 3x^2 + 3y^2 \leq 6 \iff x^2 + y^2 \leq 2 \). So,

\[
\text{vol} = \int_{x^2+y^2\leq2} \int_{4-x^2-y^2}^{10-4x^2-4y^2} dV
\]
From here it’s best to switch to cylindrical coordinates, so

\[
\text{vol} = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_{0-r^2}^{0} r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^{\sqrt{2}} r(6 - 3r^2) \, dr \, d\theta = \int_0^{2\pi} \left[ 3r^2 - \frac{3}{4}r^4 \right]_{r=0}^{r=\sqrt{2}} d\theta = 6\pi.
\]

**Problem 30:** The region \( R \) is the triangle formed by the lines \( y = x\sqrt{3} \), \( y = x \) and \( x = 2 \).

The angle made by the line \( y = x\sqrt{3} \) with the positive \( x \)-axis is \( \pi/3 \), while the angle made by the line \( y = x \) with the positive \( x \)-axis is \( \pi/4 \). The line \( x = 2 \) crosses the two lines at \((2, 2\sqrt{3})\) and \((2, 2)\).

The line \( x = 2 \) is given in polar coordinates by \( r\cos \theta = 2 \), hence \( r = \frac{2}{\cos \theta} \).

\[
\int_0^{2} \int_{x}^{x\sqrt{3}} x \, dy \, dx = \int_{0}^{\pi/3} \int_{0}^{2/\cos \theta} \cos \theta \, dr \, d\theta = \frac{8}{3} \int_{0}^{\pi/4} \frac{1}{\cos^2 \theta} \, d\theta = \frac{8}{3} \left[ \tan \theta \right]_{\theta=\pi/4}^{\theta=\pi/3} = \frac{8}{3}(\sqrt{3} - 1).
\]

**Problem 31:**

(a) The region of integration is the triangle made by the lines \( y = x \), \( y = 2x \) and \( x = 1 \). It has vertices \((0,0)\), \((1,1)\) and \((1,2)\).

(b) For \( 0 \leq y \leq 1 \), have \( y/2 \leq x \leq y \) and for \( 1 \leq y \leq 2 \), have \( y/2 \leq x \leq 1 \). So

\[
\int_0^{1} \int_{x}^{2x} dy \, dx = \int_0^{1} \int_{y/2}^{y} dx \, dy + \int_1^{2} \int_{y/2}^{1} dx \, dy.
\]