Math 202D Midterm #1 Formulas

1. \[ \frac{dy}{dt} = ay - b \Rightarrow y = Ke^{at} + \frac{b}{a} \]

2. \[ \frac{dy}{dt} + p(t)y = g(t) \Rightarrow \mu(t) = e^{\int p(t) dt}, \; y = \int \mu(t) g(t) dt \]
   \[ \text{may be an implicit soln.} \]

3. \[ \frac{dy}{dt} = g(y)h(x) \Rightarrow \int \frac{1}{g(y)} dy = \int h(x) dx \]

4. Existence & Uniqueness:
   - 1st order Linear O.D.E.
   - 1st order O.D.E.
   - 2nd order Linear O.D.E.

5. Autonomous Equations:
   - equilibrium solns (stable/unstable)
   - are zeros of f(y).

   Use this to draw a phase diagram

6. Exact Equation
   \[ M(x,y) + N(x,y) \frac{dy}{dx} = 0 \]

   This is exact \[ \iff M = 4x, N = 4y \text{ some } 4 = 4(x, y) \]
   \[ \iff My = Nx \text{ (by Thm)}. \]

   - Solns given implicitly by \[ 4(x, y) = C. \]

   - Integrating Factors to make eqn exact:
     \[ \text{If } \mu = \mu(x), \; \mu_x = \mu \left( \frac{My-Nx}{N} \right) \]
     \[ \text{If } \mu = \mu(y), \; \mu_y = \mu \left( \frac{Nx-My}{M} \right) \]
7. $ay'' + by' + cy = 0$

If $r_1, r_2$ are distinct, real zeros of $ar^2 + br + c = 0$, the the general sol'n is of the form

$$y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$$

8. **Thm** Suppose $y_1, y_2$ are two sol'n's to

$$y'' + p(t)y' + q(t)y = 0$$
on an interval $(x_1, b)$ upon which $p(t), q(t)$ are both continuous.

(i) All sol'n's are of the form

$$y = c_1 y_1 + c_2 y_2$$

for some $t_0 \in (x_1, b)$.

(ii) $W(t) =ce^{\int p(t) dt}$ for some constant $C$.

9. Turn word problems into diff. eqns.

(see §2.3)