Math 20A, Summer, 2006, Lecture 12

Web 1. What is $H(10)$ if $y = H(x)$ is continuous on $[2, 10]$, $H(2) = 50$, and $r = H'(x)$ is a step function with $H'(x) = -10$ for $2 < x < 7$, and $H'(x) = 20$ for $7 < x < 10$?

Answer: 60

2. Calculate the right Riemann sum for $\int_{0}^{2} (4x - x^2) \, dx$ corresponding to the partition of $[0, 2]$ into four equal subintervals. Draw the curve $y = 4x - x^2$ and the rectangles whose area equals the Riemann sum.

Answer: Figure A2  •  6.25

Figure A2

3. Use the Riemann sum program to predict the value of $\int_{0}^{2} (4x - x^2) \, dx$. Use the window $0 \leq x \leq 2, -1 \leq y \leq 5$.

Answer: Likely prediction: The integral equals $5 \frac{1}{3}$.

4. What is $\int_{-7}^{7} [6f(x) + 3g(x)] \, dx$ if $\int_{-7}^{7} f(x) \, dx = 4$ and $\int_{-7}^{7} g(x) \, dx = -5$?

Answer: 9

5. Perform the integration $\int (9x^8 - 8x^7) \, dx$.

Answer: $x^9 - x^8 + C$

Web 6. Evaluate $\int_{1}^{2} \left( \frac{1}{t^2} - \frac{t^2}{4} \right) \, dt$.

Answer: $-\frac{1}{12}$

7. What is the area of the region between $y = 1 + \frac{1}{4}x^2$ and the $x$-axis for $-3 \leq x \leq 3$?

Answer: $21 \frac{1}{7}$

8. The barometric pressure in a town is 745 millimeters of mercury at time $t = 1$ (hours) and its rate of change is $2t - 6$ millimeters of mercury per hour at time $t$ for $0 \leq t \leq 4$. Give a formula for the barometric pressure $P(t)$ for $0 \leq t \leq 4$.

Answer: $P(t) = t^2 - 6t + 750$ millimeters of mercury

9. Evaluate $\int_{1}^{6} \frac{1}{x} \, dx$.

Answer: $\ln(6)$

10. Find the value of $\int_{-6}^{-1} \frac{1}{x} \, dx$.

Answer: $-\ln(6)$
11. Evaluate \( \int_{-4}^{4} e^x \, dx \).
   Answer: \( e^4 - e^{-4} \)

12. Find the antiderivatives \( \int (x + e^x) \, dx \).
    Answer: \( \frac{1}{2}x^2 + e^x + C \)

13. Evaluate \( \int_{0}^{2} (6x - 2\cos x) \, dx \)
    Answer: \( 12 - 2\sin(2) \)

14. Find the antiderivatives \( \int e^t \cos(e^t) \, dt \).
    Answer: \( \sin(e^t) + C \)

15. Evaluate \( \int_{0}^{0.5} \sec x \tan x \, dx \).
    Answer: \( \sec(0.5) - 1 \)

16. Find the value of \( \int_{0}^{1} \frac{1}{\sqrt{4 - x^2}} \, dx \).
    Answer: \( \frac{1}{2} \pi \)

17. Perform the integration \( \int \frac{1}{25 + x^2} \, dx \).
    Answer: \( \frac{1}{5} \tan^{-1} \left( \frac{x}{5} \right) + C \)

18. Find the area between the \( x \)-axis and \( y = \frac{1}{1 + x^2} \) for \( 0 \leq x \leq 1 \).
    Answer: \( \tan^{-1}(1) = \frac{\pi}{4} \)