Math 20A, Summer, 2006, Lecture 5

1. Find \( \frac{d}{dx}[(5x^3 + 2x^2 - 4)(x^7 - 2x^5)] \).
   
   Answer: \( (5x^3 + 2x^2 - 4)(7x^6 - 10x^4) + (x^7 - 2x^5)(15x^2 + 4x) \)

2. What is \( \frac{dy}{dx} \) for \( y = \frac{\sqrt{x}}{x^5 + 1} \)?
   
   Answer: \( \frac{(x^5 + 1)(\frac{1}{2}x^{-1/2}) - x^{1/2}(5x^4)}{(x^5 + 1)^2} \)

3. Find \( P'(5) \) where \( P(x) = R(x)S(x) \), \( R(5) = 3 \), \( S(5) = 4 \), \( R'(5) = -3 \), and \( S'(5) = 10 \)
   
   Answer: 18

4. What is \( W'(4) \) if \( W(x) = Y(x)Z(x) \), \( Y(4) = 2 \), \( Z(4) = 5 \), \( Y'(4) = 3 \), and \( Z'(4) = 6 \)
   
   Answer: 3

5. Give an equation of the tangent line to \( y = (x^2 + x^3 + x^4)(x^5 + x^6 + x^7) \) at \( x = 1 \).
   
   Answer: Tangent line: \( y = 9 + 81(x - 1) \)

6. Figures 1 and 2 give the graphs of differentiable functions \( y = A(x) \) and \( y = B(x) \). Give approximate values of \( AB, A/B \), and of their first derivatives at \( x = 2 \).

   FIGURE 1
   
   Answer: \( A(2) \approx 2.5 \)  \( A'(2) \approx -0.6 \)  \( B(2) \approx 1.5 \)  \( B'(2) \approx 1 \)  \( [AB]_{x=2} \approx 3.75 \)  \( \left[ \frac{A}{B} \right]_{x=2} \approx 1.7 \)

   \[ \left[ \frac{d}{dx} \left( \frac{AB}{B} \right) \right]_{x=2} \approx 1.6 \]  \( \left[ \frac{d}{dx} \left( \frac{A}{B} \right) \right]_{x=2} \approx -1.5 \)

7. Find \( \frac{d}{dx}[(1 + 3x - x^2)(x^2 - 5)] \).

   Answer: \( (1 + 3x - x^2)(2x) + (x^2 - 5)(3 - 2x) \)

8. What is \( \frac{d}{dx} \left( \frac{x^2 + x}{x^2 - 2} \right) \)?

   Answer: \( -\frac{x^2 - 4x - 2}{(x^2 - 2)^2} \)

9. What is \( f'(10) \) if \( f(x) = g(x)h(x) \), \( g(10) = -4 \), \( h(10) = 560 \), \( g'(10) = 0 \), and \( h'(10) = 35 \)?

   Answer: -140

10. Find \( R'(1) \) where \( R(s) = \frac{P(s)}{Q(s)} \), \( P(1) = 13 \), \( Q(1) = -2 \), \( P'(1) = 7 \), and \( Q'(1) = -4 \).

    Answer: \( \frac{19}{2} \)
11. Give an equation of the tangent line to \( y = (1 + x - x^4)(1 - x + x^3) \) at \( x = 1 \). Then generate the curve and tangent line on your calculator.

**Answer:** Tangent line: \( y = 1 - (x - 1) \)  - Figure A1

![Figure A1](image)

12. At the beginning of 1991 there were 2.1 million farms in the United States with an average size of 467 acres per farm; the number of farms was decreasing 0.035 million farms per year; and the average size was increasing 7 acres per farm per year. What was the total acreage of farms and at what rate was it increasing or decreasing at the beginning of 1991?

**Answer:** [Total acreage] = 980.7 million acres  - The total acreage was decreasing 1.645 million acres per year.

13. Figures 3 and 4 give the number \( N = N(t) \) (millions) of MasterCard and Visa accounts and the total outstanding debt \( D = D(t) \) (million dollars) in the U.S. as functions of the year. What were (a) the approximate average debt per credit card and (b) the rate of change with respect to time of the average debt per credit card at the beginning of 1988?

![FIGURE 3](image)  ![FIGURE 4](image)

**Answer:** (a) [Average debt per card] \( \approx 960 \) dollars per credit card  - (b) [The rate of change with respect to time of the average debt per credit card at the beginning of 1988] \( \approx 102.40 \) dollars per credit card per year

14. Find \( \frac{d}{dx}[(3x^2 + x)^5] \)

**Answer:** \( 5(3x^2 + x)^4(6x + 1) \)

15. What is \( \frac{d}{dx}\sqrt{3x + 5} \)?

**Answer:** \( \frac{3}{2}(3x + 5)^{-1/2} \)

16. \( G'(3) \) where \( G(x) = [y(x)]^{7/2} \), \( y(3) = 6 \), and \( y'(3) = -5 \)

**Answer:** \( G'(x) = -630\sqrt{6} \)
17. Give an equation of the tangent line to \( y = (x^4 + 2)^3 \) at \( x = 1 \). Then generate the tangent line with the curve in the window \(-1.4 \leq x \leq 1.4, -10 \leq y \leq 75\).

**Answer:** Tangent line: \( y = 27 + 108(x - 1) \)  ● Figure A17

18. Figure 4 shows the graph of a differentiable function \( y = U(x) \). Find approximate values of
(a) \( U \), (b) \( \frac{dU}{dx} \), (c) \( \frac{d}{dx}(xU) \), and (d) \( \frac{d}{dx}(U^3) \) at \( x = 2 \).

**Answer:** Possible answers (a) \( U(2) \approx 30 \) (b) \( U'(2) \approx 18 \) (c) \( \left[ \frac{d}{dx}(xU) \right]_{x=2} \approx 66 \)
(d) \( \left[ \frac{d}{dx}(U^3) \right]_{x=2} \approx 48,600 \)

19. What is the rate of change with respect to time of the volume \( V = w^3 \) of an expanding cubic crystal at a moment when its width is 10 millimeters and its width is increasing 3 millimeters per day?

**Answer:** 900 cubic millimeters per day