Math 20A, Summer, 2006, Lecture 6

1. Find the open intervals on which \( y = 1/x^2 \) is increasing and decreasing and on which its graph is concave up and concave down.

   **Answer:** Increasing on \((0, \infty)\) • Decreasing on \((-\infty, 0)\) • Concave up on \((-\infty, 0)\) and \((0, \infty)\)

2. Use the formulas for \( y = x^4 - 12x \) and its first and second derivatives to sketch its graph. Find the open intervals on which it is increasing and decreasing and on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

   **Answer:** Decreasing on \((-\infty, 3\sqrt[3]{3})\) • Increasing in \((3\sqrt[3]{3}, \infty)\) • Global minimum of \(y(\sqrt[3]{3}) = -9\sqrt[3]{3} \approx -13.0\) at \(x = \sqrt[3]{3} \approx 1.44\). • Concave up on \((-\infty, \infty)\) • Figure A2

![Figure A2](image1)

3. Use the formulas for \( y = 2x^3 - x^4 \) and its first and second derivatives to sketch its graph. Find the open intervals on which it is increasing and decreasing and on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

   **Answer:** Increasing on \((-\infty, 0)\) and \((0, \frac{9}{2})\) [or on \((-\infty, \frac{3}{2})\)] • Decreasing on \((\frac{9}{2}, \infty)\) • Global maximum of \(\frac{27}{16}\) at \(x = \frac{3}{2}\) • Concave up on \((0, 1)\) • Concave down on \((-\infty, 0)\) and \((1, \infty)\) • Inflection points: \((0, 0)\) and \((1, \frac{19}{16})\) • Figure A3

![Figure A3](image2)

4. Use the formulas for \( y = 3x^2 - x^{-2} + 5 \) and its first and second derivatives to sketch its graph. Find the open intervals on which it is increasing and decreasing and on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

   **Answer:** Defined for \(x \neq 0\) • Decreasing on \((-\infty, 0)\) • Increasing on \((0, \infty)\) • Concave up for \(x < -1\) and for \(x > 1\) • Concave down for \(-1 < x < 0\) and for \(0 < x < 1\) • Inflection points: \((-1, 7)\) and \((1, 7)\) • Figure A4

![Figure A4](image3)
5. Use the formulas for \( f(x) = 2x^{1/2} - x \) and its first and second derivatives to sketch its graph. Find the open intervals on which it is increasing and decreasing and on which its graph is concave up and concave down. Find any local or global maxima and minima and inflection points.

**Answer:** Defined for \( x \geq 0 \) • Increasing on \((0, 1)\) • Decreasing on \((1, \infty)\) • Global maximum: 1 at \( x = 1 \) • Concave down for \( x > 0 \) • Figure A5

![Figure A5](image)

6. The graph of the derivative \( y = h'(x) \) of a function \( y = h(x) \) is shown in Figure 1. (a) On what open intervals is the function increasing and decreasing? (b) Where does it have local maxima and minima? (c) What are the open intervals on which the graph \( y = h(x) \) is concave up and concave down? (d) Locate any inflection points of the graph.

**Answer:** (a) Decreasing on \((-2, 0)\) and \((6, 2)\) • Increasing on \((0, 6)\) (b) Local minimum at \( x = 0 \) • Local maximum at \( x = 6 \) (c) Concave up on \((-2, 3)\) • Concave down on \((3, 8)\) (d) Inflection point at \( x = 3 \)

![Figure 1](image)

7. Figure 2 shows the graph of the second derivative of a function \( y = g(x) \). (a) What are the open intervals in which the graph of \( y = g(x) \) is concave up and on which it is concave down? (b) At what values of \( x \) does the graph of \( y = g(x) \) have inflection points?

**Answer:** (a) Concave down on \((-\infty, -1)\) and \((4, \infty)\) • Concave up on \((-1, 4)\) (b) Inflection points at \( x = -1 \) and \( x = 4 \)

![Figure 2](image)