1. Find the maximum and minimum value of \( y = x^3 - 3x + 5 \) for \( 0 \leq x \leq 2 \).

   \textbf{Answer:} [Maximum] = \( y(2) = 7 \) \bullet [Minimum] = \( y(1) = 3 \)

2. Find the maximum and minimum value of \( y = x + 4/x^2 \) for \( x > 0 \).

   \textbf{Answer:} [Minimum] = \( y(2) = 3 \) \bullet There is no maximum.

3. Find the maximum and minimum value of \( y = 6x^4 - 4x^6 \) for \( -2 \leq x \leq 2 \).

   \textbf{Answer:} [Maximum] = \( y(\pm 1) = 2 \) \bullet [Minimum] = \( y(\pm 2) = -160 \)

4. You want to make a rectangular garden with as large an area as possible using a wall as one side and 40 feet of fence for the three other sides (Figure 1). Find the dimensions for which the area is greatest.

   \textbf{Answer:} The area is a maximum for \( w = 10 \) feet and \( L = 20 \) feet. \bullet The maximum area is 200 square feet.

5. A rectangular box with a square base is to be constructed so that it has a volume of 20 cubic meters. The material for the base and top costs 4 dollars per square meter and the material for the four sides costs 2 dollars per square meter. How wide and how tall should the box be to minimize the cost of the material?

   \textbf{Answer:} The box that costs the least has width \( w = 10^{1/3} \approx 2.154 \) meters and height \( h = 20/w^2 = 2(10^{1/3}) \approx 4.309 \) meters.

6. Find the point on the right half of the curve \( y = 1/x \) that is closest to the origin.

   \textbf{Answer:} The closest point to the origin is \((1,1)\) at \( x = 1 \).

7. If you will sell \( x(p) = p/(p^3 + 4) \) units of a product if you charge \( p \) dollars per unit, what price would maximize your revenue?

   \textbf{Answer:} The revenue is a maximized if the price is $2 per item.

8. Find \( A'(6) \) where \( A(x) = (B \circ y)(x) \), \( y(6) = 7 \), \( y'(6) = 8 \), and \( B'(7) = 9 \).

   \textbf{Answer:} \( A'(6) = B'(y(6))y'(6) = B'(7)y'(6) = (9)(8) = 72 \)

9. What is \( W'(100) \) if \( W(y) = U(V(y)) \), \( V(100) = -3 \), \( V'(100) = 7 \), and \( U'(-3) = 10 \)?

   \textbf{Answer:} \( W'(100) = U'(V(100))V'(100) = U'(-3)V'(100) = (10)(7) = 70 \)

10. Find \( H'(2) \) where \( H(x) = F(x^3) \) and \( F'(8) = 10 \).

    \textbf{Answer:} \( H'(2) = 120 \)

11. After you have driven 100 kilometers, your have 75 liters of gasoline in your car, your car is consuming gasoline at the rate of 0.2 liters per kilometer, and you are traveling 80 kilometers per hour. (a) At what rate, measured in liters per hour, are you using gasoline at that moment? (b) Approximately how much gasoline do you have in your car a quarter hour later?

    \textbf{Answer:} You are using gasoline at the rate of 16 liters per hour. (b) A quarter hour later you have about 71 gallons of gas.

12. At 12:00 PM a balloon is 200 meters above the ground, it has a volume of 5 liters, it is rising 3 meters per second, and its volume is increasing at the rate of 0.001 liters per meter of height. (a) At what rate is its volume increasing with respect to time at 12:00 PM? (b) Approximately how high is the balloon and what is its approximate volume 15 seconds after 12:00 PM?

    \textbf{Answer:} (a) 0.003 liters per second (b) \( h \approx 290 + 3(15) = 245 \) meters and \( V \approx 5 + 0.003(15) = 5.045 \) liters