Problem 1  (a) (6%) Sketch the graph of the function \( f \) defined by

\[
    f(x) = \begin{cases} 
    -x^2 & \text{for } x < 1 \\
    2 & \text{at } x = 1 \\
    1/x & \text{for } x > 1.
    \end{cases}
\]

Find (3% each) (b) \( \lim_{x \to -\infty} f(x) \), (c) \( \lim_{x \to 0} f(x) \), (d) \( \lim_{x \to 1^-} f(x) \), (e) \( \lim_{x \to 1} f(x) \), and (f) \( \lim_{x \to \infty} f(x) \) if they are defined.

(g) (3%) What are the largest intervals on which \( f \) is continuous?

(h) (3%) Find all solutions \( x \) of \( f(x) = -4 \).

Problem 2  (6 points) Use the definition of the derivative to find \( g'(2) \) for \( g(x) = \frac{4}{x} \).

Problem 3  (20 points) Find the derivatives of (a) \( y = x^{3/2} + x \), (b) \( y = \sqrt{\ln x + 4} \), (c) \( y = xe^{-x} \) and (d) \( y = (x^2 + 3x)^4 \).

Problem 4  (8 points) Give an equation of the tangent line to \( y = \frac{1}{\sqrt{x}} \) at \( x = 1 \). Generate the curve and tangent line in the window \(-1 \leq x \leq 4, -1 \leq y \leq 4\) and copy them on your paper.

Problem 5  (9 points) Figure 1 shows the graph of the width \( w = w(t) \) (yards) of a square as a function of the time \( t \) (minutes). What is the approximate rate of change of the area of the square with respect to \( t \) at \( t = 30 \)?

![Figure 1](attachment:figure1.png)

Problem 6  (10 points) Find the maximum and minimum of \( y = x + \frac{4}{x} \) for \( 1 \leq x \leq 8 \).

Problem 7  (10 points) Sketch the graph of \( y = x^3 - 12x + 7 \). Show where the function is increasing and decreasing and where its graph is concave up and concave down. Also find any local or global maxima and minima and inflection points.

Problem 8  (10 points) A rectangular box with a square base and no top is to be constructed to have a volume of one-half cubic meter. What dimensions should it have to minimize the total area of the bottom and four sides?

Scores:

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