Section 1.2. Exponential functions†

Example 1  What is the growth factor from 3 to 12?
Answer: The growth factor is 4.

Example 2  What is the growth factor from 20 to 4?
Answer: The growth factor is \(\frac{1}{5}\).

Example 3  The function \(y = h(x)\) is an exponential function that has the value 7 at \(x = 0\) and grows by the factor of 10 when \(x\) is increased by 4. Give a formula for it.
Answer: \(h(x) = 7(10^{x/4})\)

Example 4  Radium-226 has a half-life of 1620 years. If a sample has a mass of 4 grams now, what will its mass be in 1620 years? In 3240 years? In 5000 years?
Answer: [Mass in 1620 years] = 2 grams  •  [Mass in 3240 years] = 1 gram  •  [Mass in 5000 years] = \(4 \left(\frac{1}{2}\right)^{5000/1620}\) \(= 0.47\) grams. (The graph of the mass is in Figure A4.)

Example 5  Draw the graph of \(y = 5 + 3(2^x)\).
Answer: Figure A5

†Lecture notes to accompany Section 1.2 of Calculus by Hughes-Hallett et al.
Example 6  
Draw the graph of \( y = -5 - 3(2^{-x}) \).

Answer: Figure A6

\[
\begin{array}{c|c|c|c|c|c|}
-3 & -2 & -1 & 1 & 2 \\
\hline
 & & & & \\
 & & & & \\
 & & & & \\
 & & & & \\
\end{array}
\]
\[
y = -5 - 3(2^{-x})
\]
\[
\begin{array}{c|c|c|c|c|c|}
 & & & & & \\
 & & & & & \\
 & & & & & \\
 & & & & & \\
\end{array}
\]

Figure A6

Example 7  
(a) Simplify the expression \( y = \frac{\sqrt[3]{b}}{\sqrt[6]{b}} \) without using fractional or negative exponents by taking the sixth power of both sides, simplifying, and taking the sixth root. (b) Simplify the same expression by using laws of exponents.

Answer: (a) \( b = \sqrt[6]{b} \)  
(b) \( b = \sqrt[6]{b} \)

Example 8  
Figure 1 shows the graphs of \( y = 1.5^x, y = e^x, \) and \( y = 6^x \). (a) Which is the upper curve, which is the middle curve, and which is the lower curve for \( x > 0 \)? (b) Which is the upper curve, which is the middle curve, and which is the lower curve for \( x < 0 \)?

Answer: (a) For \( x > 0 \), \( y = 1.5^x \) is the lower curve, \( y = e^x \) is the middle curve, and \( y = 6^x \) is the upper curve.  
(b) For \( x < 0 \), \( y = 1.5^x \) is the upper curve, \( y = e^x \) is the middle curve, and \( y = 6^x \) is the lower curve.

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:†  
Section 0.3: Example 3  
Section 3.3: Example 1

†The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.