Section 1.5. Trigonometric functions†

Example 1  Find the cosine, sine, and tangent of the angle $\psi$ (psi) in the right triangle of Figure 1.

![Figure 1](image)

Answer: $\cos \psi = \frac{3}{\sqrt{34}}$  \hspace{1cm} $\sin \psi = \frac{5}{\sqrt{34}}$  \hspace{1cm} $\tan \psi = \frac{5}{3}$

Example 2  The largest ferris wheel of all time was designed by George Ferris and built for the 1893 World’s Columbian Exposition in Chicago. It had a 125 foot radius and could carry 2160 passengers.\(^1\) Suppose that the center of the wheel was 150 feet above the ground and that it turned counterclockwise as viewed in the schematic sketch with a uv-plane in Figure 2. The dot represents a car on Ferris’ wheel with coordinates $(u(\theta), v(\theta))$ at the angle $\theta$ (radians). (a) Which of Figures 3 and 4 shows the graph of $u = u(\theta)$ and which shows the graph of $v = v(\theta)$? (b) Give formulas for $u(\theta)$ and $v(\theta)$.

![Figure 2](image)

Answer: (a) $u = u(\theta)$ is in the second drawing. \hspace{1cm} $v = v(\theta)$ is in the first drawing.

(b) $u(\theta) = 125 \cos(\theta)$ \hspace{1cm} $v(\theta) = 150 + 125 \sin \theta$

---

†Lecture notes to accompany Section 1.5 of Calculus by Hughes-Hallett et al.

\(^{1}\)“The Ferris Wheel” by H. Petrovski, American Scientist, May-June, 1993, pp. 216–222.
Example 3  Figure 5 shows the graph of a sinusoidal function. Give its formula.

Answer: $y = 5 \sin \left( \frac{1}{2} \pi x \right)$

Example 4  Figure 6 shows a vertical pole 10 feet high and its shadow that is cast on the horizontal ground by the setting sun. Find a formula for the length $s = s(\theta)$ of the shadow as a function of the angle $\theta$ between the rays of the sun and the pole.

Answer: $s(\theta) = 10 \tan \theta$, $0 \leq \theta < \frac{1}{2} \pi$

Example 5  Figure 7 shows the graph of $s = s(\theta)$ from Example 4. Use Figure 6 (a) to explain why $s(0) = 0$, (b) to explain why $s(\theta)$ is larger for larger $\theta$, and (c) to find the value of $s \left( \frac{1}{4} \pi \right)$.

Answer: (a) $s(0) = 0$ because there is no shadow when the sun is directly over the pole. (b) $s(\theta)$ is larger for larger $\theta$ because the shadow is longer when the sun is lower in the sky. (c) $s \left( \frac{1}{4} \pi \right) = 10$ because with the sun at this angle the right triangle in Figure 6 is isosceles.
Example 6  What is the angle $\gamma$ in the right triangle of Figure 8?

\[\gamma = \sin^{-1} \left( \frac{5}{12} \right) \approx 0.42978 \text{ radians} \]

Answer: $\gamma = \sin^{-1} \left( \frac{5}{12} \right) \approx 0.42978 \text{ radians}$

Example 7  Ravenna, Italy is 73 kilometers north and 76 kilometers east of Florence (Figure 9). (a) How far is Ravenna from Florence? (b) What is the direction from Florence toward Ravenna?

\[\text{Answer: (a) [Distance]} = \sqrt{73^2 + 76^2} \approx 105 \text{ kilometers} \]

\[\text{(b) The direction from Florence to Ravenna is } \tan^{-1} \left( \frac{73}{76} \right) \approx 0.765 \text{ radians or 43.8° north of east.} \]

Interactive Examples
Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:\footnote{The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.}

Section 0.5: Examples 1 through 7