**Math 10A. Lecture Examples.**

Section 1.8. Limits†

*Example 1* Figure 1 shows the graph of the function,

\[ \begin{align*}
F(x) &= \begin{cases} 
4 - x^2 & \text{for } x < 1 \\
4 & \text{for } x = 1 \\
2x & \text{for } x > 1.
\end{cases}
\]

(a) Calculate \( F(x) \) at \( x = 0.9, 0.99, 0.999, \) and \( 0.999, 1.1, 1.01, 1.001 \) and \( 1.0001. \) (b) What is \( \lim_{x \to 1^-} F(x) \)? (c) What is \( \lim_{x \to 1^+} F(x) \)?

![Figure 1](image)

**Answer:** (a) The values are in the table below. (b) \( \lim_{x \to 1^-} F(x) = 3 \) (c) \( \lim_{x \to 1^+} F(x) = 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( F(x) = 4 - x^2 )</th>
<th>( x )</th>
<th>( F(x) = 2x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>3.19</td>
<td>1.1</td>
<td>2.2</td>
</tr>
<tr>
<td>0.99</td>
<td>3.0199</td>
<td>1.01</td>
<td>2.02</td>
</tr>
<tr>
<td>0.999</td>
<td>3.001999</td>
<td>1.001</td>
<td>2.002</td>
</tr>
<tr>
<td>0.9999</td>
<td>3.00019999</td>
<td>1.0001</td>
<td>2.0002</td>
</tr>
</tbody>
</table>

†Lecture notes to accompany Section 1.8 of Calculus by Hughes-Hallett et al.
Example 2  Figure 2 shows the graph of \( y = \sin(1/x) \) for \( x > 0 \). Explain why \( \lim_{x \to 0^+} \sin(1/x) \) does not exist.

**Answer:** Since \( \sin x \) oscillates infinitely often between 1 and \(-1\) as \( x \) increases through all positive values, \( \sin(1/x) \) oscillates infinitely often between 1 and \(-1\) and does not approach any one number as \( x \) approaches 0 from the right.

Example 3  What is \( \lim_{x \to 5^+} \left[ A(x)B(x) + \frac{C(x)}{D(x)} \right] \) if \( \lim_{x \to 5^+} A(x) = 2, \lim_{x \to 5^+} B(x) = 5, \lim_{x \to 5^+} C(x) = 6, \) and \( \lim_{x \to 5^+} D(x) = 3? \)

**Answer:** \( \lim_{x \to 5^+} \left[ A(x)B(x) + \frac{C(x)}{D(x)} \right] = 12 \)
Example 4  
Draw the graph of $J(x) = \begin{cases} x^2 & \text{for } x < 2 \\ 8 - 2x & \text{for } x > 2 \end{cases}$ and find the limit of $J(x)$ as $x \to 2$.

Answer: Figure A4. $\lim_{x \to 2} J(x) = 4$

Example 5  
What is $\lim_{x \to 1} \frac{x+5}{x+2}$?

Answer: $\lim_{x \to 1} \frac{x+5}{x+2} = 2$

Example 6  
Find $\lim_{x \to 1} T(x)$ where

$$T(x) = \begin{cases} x + 1 & \text{for } x \leq 1 \\ 1/x & \text{for } x > 1 \end{cases}$$

Answer: $\lim_{x \to 1} T(x)$ is not defined (does not exist).
Example 7  Figure 3 shows the graph of a function $K$, defined by

$$K(x) = \begin{cases} x + 4 & \text{for } -2 \leq x < 1 \\ x + 1 & \text{for } 1 \leq x \leq 4. \end{cases}$$

(a) At what values of $x$ for $-2 \leq x \leq 4$ is $K$ continuous?  
(b) Is $K$ continuous from the left or from the right at the values of $x$ for $-2 \leq x \leq 4$ where it is not continuous?  
(c) What are the largest intervals on which $K$ is continuous?

Answer: (a) $K$ is continuous at all $x$ with $-2 < x < 1$ and $1 < x < 4$. (b) $K$ is not continuous at $-2$ (because it is not defined for $x < -2$) but is continuous from the right at $-2$. $K$ is not continuous at 1 (because the one-sided limits are different) but is continuous from the right at 1. $K$ is not continuous at 4 (because it is not defined for $x > 4$) but is continuous from the left at 4. (c) $K$ is continuous on the intervals $[-2, 1)$ and $[1, 4]$.

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

Section 1.1: Examples 1 through 4

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1When we say that one (possibly infinite) interval $I_1$ is “larger” than another interval $I_2$, we mean that $I_1$ contains $I_2$ and is not equal to $I_2$.

2The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.