Math 10A. Lecture Examples.

Section 3.3. The Product and Quotient Rules†

Example 1  Find the derivative of \( y = (x^5 + x^3 + 1)(x^2 + 2) \) at \( x = 1 \).
Answer: \( y'(1) = 30 \)

Example 2  What is the derivative of \( y = x^2 e^x \)?
Answer: \( y'(x) = (x^2 + 2x) e^x \)

Example 3  Find the rate of change of the area of a rectangle at a moment when the width is 4 meters, the height is 2 meters, the width is increasing 3 meters per hour, and the height is increasing 5 meters per hour.
Answer: [Rate of change of the area] = 26 square meters per hour

Example 4  At the beginning of 1990 the total population of the U.S was 248.7 million, of whom 51.3\% were women, the total population was increasing at the rate of 3.5 million per year, and the percentage of women was decreasing 0.04\% per year.\((1)\) At what rate was the population of women increasing at the beginning of 1990?
Answer: [Rate of change of the population of women at the beginning of 1990] \( \approx 1.70 \) million per year

Example 5  Find the derivative of \( y = \frac{e^x}{x+3} \).
Answer: \( y'(x) = \frac{d}{dx} \left( \frac{e^x}{x+3} \right) = \frac{(x+2)e^x}{(x+3)^2} \)

Example 6  Figure 1 shows the tangent line to \( y = \frac{x^3}{1-x^2} \) at \( x = 2 \). Find an equation for it.

\[ y = \frac{x^3}{1-x^2} \]

\[ x = 2 \]

\[ y = 4 \]

\[ y = 2 \]

\[ x = 4 \]

\[ \text{FIGURE 1} \]

Answer: Tangent line: \( y = -\frac{8}{3} - \frac{4}{3}(x - 2) \)

†Lecture notes to accompany Section 3.3 of Calculus by Hughes-Hallett et al.

Example 7  On September 15, 2008, the U.S. national debt was approximately 9.7 trillion dollars and was increasing at the approximate rate of 0.67 trillion dollars per year. The U.S. population was approximately 305 million and was increasing at the approximate rate of 3.4 million per year.\(^{(2)}\) Based on these estimates, what were (a) the average debt per person and (b) the rate of change of the average debt per person at that time?

**Answer:**
(a) [Average debt] = 31,800 dollars per person
(b) [Rate of change of the average debt] = 1800 dollars per person per year

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:\(^{\dagger}\)

Section 2.6: Examples 1 through 5

\(^{(2)}\)Data adapted from the U.S. Census Bureau and The U.S. Treasury Department internet sites.

\(^{\dagger}\)The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.