Section 11.3. Euler’s method†

Example 1 Figure 1 shows the slope field for the differential equation,
\[
\frac{dy}{dx} = (1 - x)y.
\]

Draw the graph of approximate solution \( y = y_E(x) \) for \( 0 \leq x \leq 4 \) with the initial value \( y(0) = 1 \) that is obtained by Euler’s method with the partition, \( 0 < 1 < 2 < 3 < 4 \).

Answer: Figure A1a  •  (Figure A1b shows the graph of the exact solution \( y = y(x) \) with the Euler approximation.)

†Lecture notes to accompany Section 11.3 of Calculus by Hughes-Hallett et al.
Example 2  
Use Euler’s method with four subintervals to find the values at the partition points of an approximate solution of \( \frac{dy}{dx} = \frac{(1 - x)y}{x} \), \( y(1) = 3 \), \( 1 \leq x \leq 5 \). Then draw its graph.

Answer: \( y_E(1) = 3 \)  
\( y_E(2) = 1.5 \)  
\( y_E(4) = 0.5 \)  
\( y_E(5) = 0.125 \)  
Figure A2

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

† Section 9.4: Example 1

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† The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.