Section 8.3. Area and arc length in polar coordinates

Example 1 Sketch the curve in an xy-plane with the polar equation 
\[ r = 1 + \cos \theta, \quad 0 \leq \theta \leq 2\pi. \] (The curve is called a cardioid because of its heart-like shape.)

Answer: Figures A1a, A1b, and A1c

Example 2 Find the area of the region bounded by the cardioid from Example 1 with polar equation 
\[ r = 1 + \cos \theta, \quad 0 \leq \theta \leq 2\pi. \] (Use the identity \( \cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)] \) in the integration.)

Answer: \[ \text{Area} = \frac{3}{2} \pi \]

Example 3 Use polar coordinates to find the area bounded by the circle 
\[ r = 4 \sin \theta, \quad 0 \leq \theta \leq \pi \] in Figure 1.

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/.

Section 11.3: Examples 1–5

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1 Lecture notes to accompany Section 8.3 of Calculus by Hughes-Hallett et al.
2 The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.