Section 9.1. Sequences

Example 1  A piece of meat at 30°C is put in a freezer at time \( n = 0 \). The temperature of the freezer is 0°C, and the temperature of the meat \( n \) hours later is \( T = \frac{30}{n+1} \) (Figure 1).

Does the sequence \( \left\{ \frac{30}{n+1} \right\}_0^\infty \) as \( n \to \infty \) converge? If so, what is its limit?

Answer: \( \lim_{n \to \infty} \frac{30}{n+1} = 0 \) converges and its limit is 0. (The temperature of the meat approaches the temperature of the freezer as \( n \to \infty \).)

Example 2  Figure 2 shows the graph of the population \( P = 1000 \left(\frac{2^n}{6}\right) \) on day \( n \) of a colony of bacteria that consists of 1000 bacteria at \( n = 0 \).

(a) How long does it take for the population to double? (b) Does the sequence \( \left\{ 1000 \left(\frac{2^n}{6}\right) \right\}_n^\infty \) converge?

Answer: (a) The population doubles every 6 days. (b) \( \left\{ 1000 \left(\frac{2^n}{6}\right) \right\}_n^\infty \) diverges.

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†Lecture notes to accompany Section 9.1 of Calculus by Hughes-Hallett et al.
Example 3  Figure 3 shows the graph of the the number of days \( y = d_n \) in February of year \( n \geq 2000 \): \( d_n \) is 29 for leap years \( n \) when \( n/4 \) is an integer and is 28 other years. What happens to the sequence \( \{d_n\}_{n=2000}^\infty \) as \( n \to \infty \)?

![FIGURE 3](image)

Answer: \( \{d_n\}_{n=2000}^\infty \) diverges.

Example 4  Does the sequence \( \{e^{1/\sqrt{n}}\}_{n=1}^\infty \) converge or diverge? If it converges, give its limit.

Answer: The sequence converges and its limit is 1. (The table below shows that the limit is approached relatively slowly.)

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
<th>10,000</th>
<th>100,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^{1/\sqrt{n}} )</td>
<td>2.7183</td>
<td>1.3719</td>
<td>1.1052</td>
<td>1.0321</td>
<td>1.0101</td>
<td>1.0010</td>
</tr>
</tbody>
</table>

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

Section 10.1: Examples 1–5

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\(^{\dagger}\)The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.