Math 10B. Lecture Examples.

Section 9.3. Convergence of series†

Example 1 Does \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \) converge or diverge?

Answer: \( \int_{2}^{\infty} \frac{1}{x(\ln x)^2} \, dx = \frac{1}{\ln(2)} \cdot \) The improper integral and the infinite series converge. (The first 24 partial sums of the series are plotted in Figure A1.)

\[ s_N = \sum_{n=2}^{N} \frac{1}{n(\ln n)^2} \]

Figure A1

Example 2 Does \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \) converge or diverge?

Answer: \( \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx = \infty \cdot \) The infinite series diverges. (Its first 25 partial sums are plotted in Figure A2.)

\[ s_N = \sum_{n=1}^{N} \frac{1}{\sqrt{n}} \]

Figure A2

Example 3 Does \( \sum_{n=1}^{\infty} ne^{-n^2} \) converge?

Answer: \( \int_{1}^{\infty} xe^{-x^2} \, dx = \frac{1}{2}e^{-1} \cdot \sum_{n=1}^{\infty} ne^{-n^2} \) converges.

Example 4 Does \( \sum_{n=1}^{\infty} \frac{1}{n^{1.75}} \) converge or diverge?

Answer: \( \sum_{n=1}^{\infty} \frac{1}{n^{1.75}} \) converges.

†Lecture notes to accompany Section 9.3 of Calculus by Hughes-Hallett et al
Example 5  

Does \( \sum_{n=1}^{\infty} \frac{n}{n+1} \) converge or diverge?

Answer: \( \sum_{n=1}^{\infty} \frac{n}{n+1} \) diverges.

Interactive Examples

Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:

Section 10.3: Examples 1–4

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\(^\dagger\)The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.