Section 9.4. Tests for convergence

Example 1  Does the series $\sum_{n=0}^{\infty} \frac{(0.6)^n}{n+1}$ converge?

Answer: $\sum_{n=0}^{\infty} \frac{(0.6)^n}{n+1}$ converges by the Comparison Test with the convergent Geometric Series $\sum_{n=0}^{\infty} (0.6)^n$.

(All but the first partial sum of $\sum_{n=0}^{\infty} \frac{(0.6)^n}{n+1}$ in Figure A1a is less than the corresponding partial sum of $\sum_{n=0}^{\infty} (0.6)^n$ in Figure A1b.)

Example 2  Does $\sum_{n=1}^{\infty} \frac{10 \cos(3n)}{n^{3/2}}$ converge?

Answer: $\sum_{n=1}^{\infty} \frac{10 \cos(3n)}{n^{3/2}}$ converges.

Example 3  Does $\sum_{n=2}^{\infty} \frac{1}{n-1}$ converge or diverge?

Answer: $\sum_{n=1}^{\infty} \frac{1}{n-1}$ diverges.

Example 4  Does $\sum_{n=0}^{\infty} \frac{2^n + 10}{5^n}$ converge or diverge?

Answer: $\sum_{n=0}^{\infty} \frac{2^n + 10}{5^n}$ converges by the Limit Comparison Test with the convergent geometric series $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$.

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†Lecture notes to accompany Section 9.4 of Calculus by Hughes-Hallett et al
Example 5  Apply the Ratio Test to $\sum_{n=0}^{\infty} \frac{5^n}{n!}$.
Answer: The series converges.

Example 6  Apply the Ratio Test to $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3}$.
Answer: $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3}$ diverges.

Example 7  Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ converges.
Answer: The series converges by the Alternating Series Test.

Example 8  Does $\sum_{n=1}^{\infty} (-1)^n e^{1/n}$ converge?
Answer: No, the series diverges.

Interactive Examples
Work the following Interactive Examples on Shenk’s web page, http://www.math.ucsd.edu/~ashenk/:
Section 10.4: Examples 1–5
Section 10.5: Examples 1–5

†The chapter and section numbers on Shenk’s web site refer to his calculus manuscript and not to the chapters and sections of the textbook for the course.