1. The definition of complex differentiable $f(z)$. Examples, $p(z), e^z, e^{p(z)}, 1/z, 1/p(z)$ etc.

2. Key points of $e^z$ are $d/dz e^z = e^z$ and $e^z e^w = e^{z+w}$.

3. All of the usual derivative formulas hold, in particular product, sum, and chain rules:
   $$\frac{d}{dz} f(g(z)) = f'(g(z)) g'(z)$$
   and
   $$\frac{d}{dt} f(z(t)) = f'(z(t)) \dot{z}(t).$$

where $f, g$ are analytic and $z(t) \in \mathbb{C}$ is continuously differentiable.

4. Re $z, \text{Im} z, \bar{z}$, are nice functions from the real - variables point of view but are not complex differentiable.

5. The Cauchy Riemann equations hold,
   $$f_y = i f_x$$
   if $f$ is complex differentiable. Conversely, if the C.R. equations hold and the partial derivatives are continuous near some point $z$ then $f'(z)$ exists and $f'(z) = f_x(z) = -i f_y(z)$.

6. $e^z = e^x (\cos y + i \sin y)$ and $|e^z| = e^x \leq e^{|z|}$.

7. $\arg(z) = \{ \theta \in \mathbb{R} : z = |z| e^{i \theta} \}$ and $\text{Arg}(z) = \theta$ if $-\pi < \theta \leq \pi$ and $z = |z| e^{i \theta}$. Notice that $z = |z| e^{i \arg(z)}$.

8. $\log z = \text{ln} |z| + i \text{arg} z$ and its branches. I will denote a typical branch of log by $\ell$. Recall that $\ell'(z) = 1/z$ for all branches, $\ell$, of log.

9. $z^{1/n} = \sqrt[n]{|z| e^{i \arg(z)/n}}$ - a branch of $z^{1/n}$ is $z^{1/n} := e^{\frac{i}{n}(\ell)}$.

10. More generally if $c \in \mathbb{C}$ we set $z^c := e^{c \log(z)}$ and if $\ell$ is a branch of log, the we define $z^\ell := e^{\ell \log(z)}$ to be a branch of $z^c$. With this notation we have
    $$\frac{d}{dz} z^\ell = cz^{\ell-1}.$$

11. Be familiar with the following analytic functions;
    a) $\sin(z) := \frac{e^{iz} - e^{-iz}}{2i} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1}$
    b) $\cos(z) := \frac{e^{iz} + e^{-iz}}{2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{2n}$
    c) $\sinh(z) := \frac{e^z - e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} z^{2n+1}$
    d) $\cosh(z) := \frac{e^z + e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n)!} z^{2n}$
    e) $\tan(z) = \frac{\sin(z)}{\cos(z)} = \frac{-e^{iz} - e^{-iz}}{e^{iz} + e^{-iz}}$
    f) $\tanh(z) = \frac{\sinh(z)}{\cosh(z)} = \frac{e^z - e^{-z}}{e^z + e^{-z}}$

12. You should have some familiarity with some of the inverse trig. functions as well.

13. It is sometimes useful to know that derivatives of inverse functions can be found using the “converse” to the chain rule covered in class.

14. Integration:
   $$\int_a^b z(t) \, dt := \int_a^b x(t) \, dt + i \int_a^b y(t) \, dt.$$

   All of the usual integration rules hold, like the fundamental theorem of calculus, linearity and integration by parts.

15. Be able to use the Cauchy Riemann equations to check where a function $(f)$ is complex differentiable and where it is you should know that $f'(z) = f_x(z)$.

16. Be able to parametrize simple contours.

17. Be able to compute contour integrals by parametrizing the contour and then evaluating the integral using
   $$\int_C f(z) \, dz = \int_a^b f(z(t)) \dot{z}(t) \, dt$$
   where $C$ is the contour $z = z(t)$ with $a \leq t \leq b$. [Remember you are formally letting $z = z(t)$!]

18. Be able to compute contour integrals using the fundamental theorem of calculus: if $f$ is analytic on a neighborhood of a contour $C$, then
   $$\int_C f'(z) \, dz = f(C_{\text{end}}) - f(C_{\text{begin}}).$$

19. General knowledge of material from the start of the course. For example you should recall that $z^{1/n} = \sqrt[n]{|z| e^{i \arg(z)/n}}$ where $\arg(z) = \{ \theta \in \mathbb{R} : z = |z| e^{i \theta} \}$. 

Study Guide for Math 120A Exam 2 (2/20/2015) (What you should know)