HW1, #1. Do Exercise 0.5.3 (on p. 21).

HW1, #2. Let $X$ be the set of nonnegative real numbers $x$ such that $x^2 < 2$.
(a) Show that $\sup X$ exists.
(b) Let $a = \sup X$. Show that $a^2 \leq 2$.

HW1, #3. Let $\{a_n\}_{n \geq 1}$ and $\{b_n\}_{n \geq 1}$ be sequences of nonnegative real numbers such that $a_n \leq b_n$ for all $n \geq 1$. Show that if $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

HW1, #4. Do Exercise 0.7.4 (on p. 31).

HW1, #5. For each of the following subsets, state whether it is open or closed (or both or neither) and justify why.
(a) The line $y = 1$ in the $(x, y)$-plane.
(b) The $(y, z)$-plane in $\mathbb{R}^3$.
(c) The unit circle centered at $(0, 0)$ in $\mathbb{R}^2$.
(d) The set $\{(x, 0) \mid x > 0\}$ in $\mathbb{R}^2$.
(e) The set $\{(x, 0) \mid x \geq 0\}$ in $\mathbb{R}^2$.
(f) The line $y = x$ in $\mathbb{R}^2$. That is, the set $\{(x, x) \mid x \in \mathbb{R}\}$ in $\mathbb{R}^2$.
(g) The sphere of radius 2 centered at $(0, 0, 0)$ in $\mathbb{R}^3$.

HW1, #6. State whether the following limits exist, and prove it.
(a) $\lim_{(x, y) \to (0, 0)} \frac{x^3}{x^2 + y^2}$.
(b) $\lim_{(x, y) \to (0, 0)} \frac{x^2 y^2}{x^2 + y^2}$.
(c) $\lim_{(x, y) \to (0, 0)} \frac{xy}{x^2 + y^2}$.
(d) $\lim_{(x, y) \to (0, 0)} \frac{x - y}{x^2 + y^2}$.
(e) $\lim_{(x, y) \to (0, 0)} \frac{y^2 \sqrt{|x|}}{x^2 + y^2}$.
(f) $\lim_{(x, y) \to (0, 0)} \frac{x \sqrt{|y|}}{x^2 + y^2}$.