1. (§3.6, #4.) Find the general solution to the following differential equation:
\[ y'' + 2y' = 3 + 4\sin 2t. \]

Hint. You may use without proof: the general solution to the homogeneous solution \( y'' + 2y' = 0 \) is \( c_1 + c_2e^{-2t} \).

**Solution:** Method of undetermined coefficients: Let
\[
y_p = At + B\cos 2t + C\sin 2t
\]
(subscript \( p \) stands for ‘particular’ solution). Then
\[
y'_p = A - 2B\sin 2t + 2C\cos 2t,
\]
\[
y''_p = -4B\cos 2t - 4C\sin 2t.
\]

For \( y_p \) to be a solution is equivalent to:
\[
3 + 4\sin 2t = -4B\cos 2t - 4C\sin 2t + 2(A - 2B\sin 2t + 2C\cos 2t)
\]
\[
= 2A + (-4B + 4C)\cos 2t + (-4C - 4B)\sin 2t.
\]

Therefore
\[
3 = 2A,
0 = -4B + 4C,
4 = -4C - 4B.
\]

We conclude
\[
A = \frac{3}{2},
B = C = -\frac{1}{2}.
\]

So a particular solution is:
\[
y_p = \frac{3}{2}t - \frac{1}{2}\cos 2t - \frac{1}{2}\sin 2t.
\]

The general solution is:
\[
y = c_1 + c_2e^{-2t} + \frac{3}{2}t - \frac{1}{2}\cos 2t - \frac{1}{2}\sin 2t.
\]

2. (§3.2, #21.) Find the fundamental set of solutions \( y_1 \) and \( y_2 \) for the following differential equation and initial point:
\[ y'' + y' - 2y = 0, \quad t_0 = 0. \]

The solution \( y_1 \) satisfies the initial conditions:
\[
y_1 (0) = 1, \quad y'_1 (0) = 0
\]
and the solution \( y_2 \) satisfies the initial conditions:
\[
y_2 (0) = 0, \quad y'_2 (0) = 1.
\]

**Solution:** Characteristic equation is:
\[
0 = r^2 + r - 2 = (r + 2)(r - 1).
\]
Roots are $r = 1$ and $r = -2$. General solution is:

$$y = c_1 e^t + c_2 e^{-2t}.$$ 

Then

$$y' = c_1 e^t - 2c_2 e^{-2t}.$$ 

For $y_1$

$$1 = y_1(0) = c_1 + c_2,$$
$$0 = y_1'(0) = c_1 - 2c_2.$$

So $c_1 = \frac{2}{3}$ and $c_2 = \frac{1}{3}$, which imply

$$y_1 = \frac{2}{3} e^t + \frac{1}{3} e^{-2t}.$$ 

For $y_2$

$$0 = y_1(0) = c_1 + c_2,$$
$$1 = y_1'(0) = c_1 - 2c_2.$$

So $c_1 = \frac{1}{3}$ and $c_2 = -\frac{1}{3}$, which imply

$$y_1 = \frac{1}{3} e^t - \frac{1}{3} e^{-2t}.$$