1. (4 points) Let $f$ be the function defined by $f(x, y, z) = (x + y)z^2$. Let $\mathbf{F} = \nabla f$ be the gradient vector field of $f$. Evaluate the line integral $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$ over the path $\mathbf{c} : [0, 1] \rightarrow \mathbb{R}^3$ defined for all $0 \leq t \leq 1$ by

$$\mathbf{c}(t) = \left( t^2 + \sin(\pi t), \frac{t^3}{1 + t}, t^5 + \sqrt{1 + 8t} \right).$$

*Hint: Do not rush into complicated calculations....*

2. (5 points) Compute the line integral $\int_{\mathbf{C}} (x - y)dx + (x + y)dy$, where $\mathbf{C}$ is the oriented arc of the circle defined by $\mathbf{C} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1, x \geq 0, y \geq 0\}$ and oriented counter-clockwise. Be sure to sketch $\mathbf{C}$ and to indicate its orientation.

3. (5 points) Compute the surface integral $\int \int_{\mathbf{S}} f\,d\mathbf{S}$ of the function $f$ defined by $f(x, y, z) = 3x^3y$ over the surface $\mathbf{S}$ defined by $\mathbf{S} = \{(x, y, z) \in \mathbb{R}^3 : z = x^3, 0 \leq x \leq 1, 0 \leq y \leq \pi\}$.

4. (6 points) Let $\mathbf{S}$ be the ellipsoidal surface defined by

$$\mathbf{S} = \left\{ (x, y, z) \in \mathbb{R}^3 : x^2 + \left( \frac{y}{2} \right)^2 + z^2 = 1, z \geq 0 \right\},$$

and oriented with outward-pointing unit normal vectors, i.e., the unit normal vector $\mathbf{n} = (n_1, n_2, n_3)$ at each point on the surface is such that $n_3 \geq 0$. Let $\mathbf{F}$ be the vector field defined at any point $(x, y, z) \in \mathbb{R}^3$ by

$$\mathbf{F}(x, y, z) = (xz, yz, 1).$$

Evaluate the flux of the vector field $\mathbf{F}$ across $\mathbf{S}$, that is the surface integral $\int \int_{\mathbf{S}} \mathbf{F} \cdot d\mathbf{S}$.

*Hint: The surface may be parametrized by*

$$\Phi(\phi, \theta) = (\sin \phi \cos \theta, 2 \sin \phi \sin \theta, \cos \phi), \quad \text{for } (\phi, \theta) \in \left[ 0, \frac{\pi}{2} \right] \times [0, 2\pi].$$