Problem 1: Prove that a finite non-empty set of real numbers contains a minimum element.

Problem 2: Let $A$ and $B$ be finite sets of a real numbers with $A \subseteq B$. Prove that $\min B \leq \min A \leq \max A \leq \max B$.

Problem 3: Let $n$ be a positive integer. Prove that the number of ordered pairs $(A, B)$ of subsets of $\mathbb{N}_n$ which satisfy $A \subseteq B$ is $3^n$.

   Hint: Let $X$ denote the set of such ordered pairs. Try to define a bijection $\varphi : X \rightarrow \text{Fun}(\mathbb{N}_n, \mathbb{N}_3)$.

Problem 4: Let $X$ and $Y$ be disjoint (not necessarily finite!) sets and let $k$ be a positive integer. Construct a bijection $\varphi : \bigcup_{i=0}^{k} \mathcal{P}_i(X) \times \mathcal{P}_{k-i}(Y) \rightarrow \mathcal{P}_k(X \cup Y)$.

Explain what this has to do with the Vandermonde convolution:

$$\sum_{i=0}^{k} \binom{n}{i} \binom{m}{k-i} = \binom{n+m}{k}.$$ 

Problem 5: Let $X$ be a finite set with $|X| = n$ and let $0 \leq k \leq n$. Construct a bijection $\varphi : \mathcal{P}_k(X) \rightarrow \mathcal{P}_{n-k}(X)$.

Explain what this has to do with the identity $\binom{n}{k} = \binom{n}{n-k}$.

Problem 6: Let $X$ be a finite nonempty set with $|X| = n$ and let $1 \leq k \leq n$. Let $x_0 \in X$. Construct a bijection $\varphi : \mathcal{P}_k(X) \rightarrow \mathcal{P}_{k-1}(X - \{x_0\}) \cup \mathcal{P}_k(X - \{x_0\})$.

Explain what this has to do with the Pascal recursion:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$ 

Problem 7: We are given 17 points inside a regular triangle of side length 1. Prove that there are two points among them whose distance is not more than $1/4$. (Hint: Subdivide the triangle in a clever way and apply the Pigeonhole Principle.)