Math 109: Spring 2016
Practice Midterm 2

Instructions: Please write your name and section number on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and explain your reasoning. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [15] Let $f : X \to Y$ be a function between sets. Suppose there exists a function $g : Y \to X$ such that $f \circ g = I_Y$. Prove that $f$ is surjective.

Problem 2: [15] Let $X$ be a finite set with $|X| = n$ and let $0 \leq k \leq n$. Describe a bijection

$$\varphi : \mathcal{P}_k(X) \longleftrightarrow \mathcal{P}_{n-k}(X).$$

Be sure to prove that your map is actually a bijection.

Problem 3: [15] Prove or disprove: For any finite sets $X$ and $Y$ and any non-negative integer $k$ we have

$$|\mathcal{P}_k(X \times Y)| = |\mathcal{P}_k(X) \times \mathcal{P}_k(Y)|.$$

Problem 4: [15] Let $\mathbb{Q}$ be the set of rational real numbers and $\mathbb{I}$ be the set of irrational real numbers. Does there exist a bijection $\psi : \mathbb{Q} \to \mathbb{I}$? Justify your answer.

Problem 5: [20] Prove that two people on Facebook have the same number of friends.

Problem 6: [15] Give an example of a set $X$ and a proper subset $Y \subset X$ and a bijection $f : Y \to X$. 
