Problem 1: Prove that for all real numbers $a, b, c, d$ we have

$$-2ab - 2ac - 2ad - 2bc - 2bd - 2cd \leq a^2 + b^2 + c^2 + d^2.$$ 

Problem 2: Let $a$ be an integer. Prove that 0 divides $a$ if and only if $a = 0$. (Hint: This problem has two parts!)

Problem 3: Let $a, b,$ and $c$ be integers. Prove that if $a$ divides $b$ or $a$ divides $c$, then $a$ divides $bc$.

Problem 4: For this problem, we define an integer $n$ to be ‘odd’ if there is another integer $q$ such that $n = 2q + 1$. We define an integer $n$ to be ‘even’ if $n$ is not odd. Prove that if $n$ is an integer and $n^2$ is even, then $n$ is even.

Problem 5: What is wrong with the following “proof” that 1 is the largest integer? “Let $n$ be the largest integer. Then, since 1 is an integer we must have $1 \leq n$. On the other hand, since $n^2$ is also an integer we must have $n^2 \leq n$ from which it follows that $n \leq 1$ (since $n$ is positive). Thus, since $1 \leq n$ and $n \leq 1$ we must have that $n = 1$. Thus 1 is the largest integer as claimed.”

What does this argument prove?

Problem 6: Recall from class (or from the textbook) that we have the following axioms concerning inequality of real numbers.

(1) (Trichotomy Law) For any two real number $a$ and $b$, one and only one of $a < b$, $a = b$, or $a > b$ holds.

(2) (Addition Law) For any three real numbers $a, b,$ and $c$, we have that $a < b$ if and only if $a + c < b + c$.

(3) (Multiplication Law) For any three real numbers $a, b,$ and $c$, we have that $(a < b$ if and only if $ac < bc$, if $c > 0)$ and $(a < b$ if and only if $ac > bc$, if $c < 0)$.

(4) (Transitive Law) For any three real number $a, b,$ and $c$, if $a < b$ and $b < c$, then $a < c$.

Use these axioms to prove by contradiction that there is no smallest positive real number.