Math 109: Winter 2014
Homework 8
Due 5:00pm on Friday 3/14/2014

Problem 1: Use the Euclidean algorithm to find the greatest common divisor \( d \) of \( a = 165 \) and \( b = 252 \). Express \( d \) as an integer linear combination of \( a \) and \( b \).

Problem 2: Let \( n > 2 \) be an integer. Prove that if there are no non-zero integer solutions to the equation \( x^n + y^n = z^n \), then there are no non-zero rational solutions.

Problem 3: How many equivalence relations are there on a set of cardinality 4?

Problem 4: For each of the following relations on the set \( X \), determine whether it is reflexive, whether it is symmetric, and whether it is transitive. For those which are equivalence relations, describe the equivalence classes.

1. For \( X = \mathbb{Z} \), put \( a \sim b \iff ab \neq 0 \).
2. For \( X = \mathbb{Z} \), put \( a \sim b \iff ab \geq 0 \).
3. For \( X = \mathbb{Z}^+ \), put \( a \sim b \iff ab > 0 \).
4. For \( X = \mathbb{Z} - \{0\} \), put \( a \sim b \iff ab > 0 \).
5. For \( X = \mathbb{Z}^+ \), put \( a \sim b \iff ab < 0 \).
6. For \( X = \mathbb{Z} - \{0\} \), put \( a \sim b \iff ab < 0 \).

Problem 5: Which of these formulae define a (well-defined) function \( f : \mathbb{Q} \to \mathbb{Q} \)?

1. \( f(a/b) = a^2/b^2 \).
2. \( f(a/b) = a^2/b^3 \).
3. \( f(a/b) = b/a \).
4. \( f(a/b) = a + b \).
5. \( f(a/b) = (a - b)/2b \).