Problem 1: (a) “$|X| = n$” means there exists a bijection $f : \mathbb{N}_n \rightarrow X$. (b) Suppose $f : \mathbb{N}_n \rightarrow X$ and $g : \mathbb{N}_n \rightarrow Y$ are bijections. Then the function $h : \mathbb{N}_{2n} \rightarrow X \cup Y$ defined by $h(i) = \begin{cases} f(i) & 1 \leq i \leq n \\ g(i - n) & n + 1 \leq i \leq 2n \end{cases}$ is a bijection because $X$ and $Y$ are disjoint. Therefore $|X \cup Y| = 2n$.

Problem 2: Let $X$ be the set of people on Facebook. We have that $|X| = n \geq 2$. Define a function $f : X \rightarrow \{0, 1, \ldots, n-1\}$ by letting $f(x)$ be the number of friends of $x$. If $0 \in \text{Im}(f)$ or $n-1 \in \text{Im}(f)$ we are done by the pigeonhole principle. So we may assume that there exist $x, y \in X$ such that $f(x) = 0$ and $f(y) = n-1$. This means that $x$ has no friends while $y$ is friends with everyone else on Facebook. This is impossible.

Problem 3: Suppose $x \in (A \cup B) - C$. Then $x \in A$ or $x \in B$ and $x \notin C$. If $x \in A$ then $x \in A - C$, so $x \in (A - C) \cup (B - C)$. If $x \in B$ then $x \in B - C$, so $x \in (A - C) \cup (B - C)$. We conclude that $(A \cup B) - C \subseteq (A - C) \cup (B - C)$.

Suppose $x \in (A - C) \cup (B - C)$. If $x \in A - C$ then $x \in A$ and $x \notin C$, so $x \in A \cup B$ and $x \in (A \cup B) - C$. If $x \in B - C$ then $x \in B$ and $x \notin C$, so $x \in (A \cup B) - C$. We conclude that $(A \cup B) - C = (A - C) \cup (B - C)$.

Problem 4: Suppose that $f = g$. Let $(x, f(x)) \in G_f$. Then $f(x) = g(x)$, so $(x, f(x)) = (x, g(x)) \in G_g$. Therefore $G_f \subseteq G_g$. By symmetry, $G_f = G_g$.

Suppose that $G_f = G_g$. Let $x \in X$, so that $(x, f(x)) \in G_f$. Then $(x, f(x)) \in G_g$. This implies that $f(x) = g(x)$, so that $f = g$ as functions.

Problem 5: The function $f$ is not uniformly continuous if there exists $\epsilon > 0$ such that for all $\delta > 0$ there exist $x, y \in \mathbb{R}$ with $|x - y| < \delta$ but $|f(x) - f(y)| \geq \epsilon$. 