Problem 1. Write down the first 10 rows of Pascal’s triangle (see p. 151). Then find the coefficient of the monomial \(a^4b^6\) in the binomial expansion of \((a + b)^{10}\).

Problem 2. What is the coefficient of \(a^2b^{48}\) in the binomial expansion of \((a - b)^{50}\)? Is there a relation between the binomial coefficients \(\binom{n}{2}\) and the triangular numbers \(T_n = \frac{1}{2}n(n+1)\)?

Problem 3. Let \(f : \mathbb{R} \to \mathbb{R}\) be the polynomial function given by the formula
\[
f(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1.
\]
Find its roots, along with their multiplicities. Give an explicit formula for its inverse \(f^{-1}\).

Problem 4. Show that the product of any \(n\) consecutive positive integers, \(m, m + 1, m + 2, \ldots, m + n - 1\), is divisible by \(n! = 1 \cdot 2 \cdots n\).

Problem 5. Let \(m\) and \(n\) be integers such that \(0 \leq m \leq n\). Prove that
\[
\sum_{i=0}^{m} (-1)^i \binom{n}{i} \binom{n}{m-i} = \begin{cases} 
(-1)^{\frac{m}{2}} \binom{n}{\frac{m}{2}} & \text{if } m \text{ is even,} \\
0 & \text{if } m \text{ is odd.}
\end{cases}
\]

Hint: Expand \((x + 1)^n(x - 1)^n = (x^2 - 1)^n\) in two ways, and compare coefficients.

Problem 6. Let \(A_1, A_2, \ldots, A_n, \ldots\) be a sequence of countable sets. Prove that their union \(\bigcup_{n=1}^{\infty} A_n\) is countable as well. How about their intersection \(\bigcap_{n=1}^{\infty} A_n\)?

Problem 7. A number \(x \in \mathbb{R}\) is called algebraic if it satisfies a polynomial equation,
\[
x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = 0,
\]
with rational coefficients \(a_0, a_1, \ldots, a_{n-1} \in \mathbb{Q}\). Show that \(\sqrt{2 + \sqrt{3}}\) is algebraic. Prove that the set of all algebraic numbers (for varying \(n\)) is a countable set.

Problem 8. Is the power set \(\mathcal{P}(\mathbb{N})\) countable? Explain why or why not.