1. [10 points total]: Circle your answer to each of the following true/false or multiple-choice questions. [2 points each]:

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a function and $x_0 \in \mathbb{R}^n$.

(a) **True or False**: If $f$ is differentiable at $x_0$, then all of the partial derivatives of $f$ must exist at $x_0$.

(b) **True or False**: If all of the partial derivatives of $f$ exist at $x_0$, then $f$ must be differentiable at $x_0$.

(c) **True or False**: If $f$ is differentiable at $x_0$, then $f$ must be continuous at $x_0$.

(d) Which of the following double integrals are evaluated over this shaded domain?

(i) $\int_0^1 \int_x^{\sqrt{x}} dx \, dy$

(ii) $\int_0^1 \int_y^{\sqrt{y}} dx \, dy$

(iii) $\int_0^1 \int_0^{\sqrt{y}} dx \, dy$

A. (i) only
B. (ii) only
C. (iii) only
D. (i) and (ii)
E. (i) and (iii)

(e) The shaded region below is

A. $x$-simple
B. $y$-simple
C. both $x$-simple and $y$-simple
D. neither $x$-simple nor $y$-simple
2. [10 points total]:

(a) [2 points]: Let $g : \mathbb{R}^3 \to \mathbb{R}$ be defined by $g(x, y, z) = xy^2 + \cos z$. Find the matrix of partial derivatives $Dg(x, y, z)$.

(b) [3 points]: Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $f(u, v) = (u, v \sin u, e^v - u)$. Find the matrix of partial derivatives $Df(u, v)$. 
(c) [2 points]: Find $f(0, 1)$ and $Df(0, 1)$.

(d) [3 points]: Let $h : \mathbb{R}^3 \to \mathbb{R}$ be defined by $h = g \circ f$. Use the chain rule to find $Dh(0, 1)$. 
3. [10 points]: Evaluate

\[
\int_{1}^{e} \int_{1}^{\ln x} \frac{\cos(y^2)}{x} \, dy \, dx
\]
4. [10 points]: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(u, v) = (u + e^v, -2u + e^v)$. Let $D^* = [0, 1] \times [0, 1]$ in the $u$-$v$ plane. Calculate

$$\iint_{T(D^*)} [x - y] \, dA$$
5. [10 points]: One day, Bill the baker spills an entire bag of sugar on his (2 meter)-by-(2 meter) baking table, represented by $D = [-1, 1] \times [-1, 1]$. Assume the planar density of sugar over a point $(x, y)$ is given by

$$f(x, y) = x^2 + y^2 \text{ mg/m}^2$$

Luke the very lucky ant then eats his way across the table in path given by

$$c(t) = (e^{-t} \cos t, e^{-t} \sin t) \text{ from } t = 0 \text{ to } t = 10.$$ 

If the path Luke eats is 1 millimeter ($= \frac{1}{1000}$ meters) wide, then

$$[f(c(t)) \text{ mg/m}^2][\frac{1}{1000} \text{ m}] = \frac{f(c(t))}{1000} \text{ mg/m}$$

approximates the linear density of sugar along Luke’s path. In mg, about how much sugar did Luke eat?