Instructions

1. Write your name, student I.D. number, and the section you will attend to retrieve your exam on the front of your blue book.

2. Start each of the following questions on a new page (you do not need to start separate parts of a question on different pages).

3. Except for true/false questions, you must show all of your work. Any answer without supporting work will receive no credit.

You may find the following table useful:

<table>
<thead>
<tr>
<th>θ</th>
<th>cos(θ)</th>
<th>sin(θ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>π/4</td>
<td>\frac{\sqrt{2}}{2}</td>
<td>\frac{\sqrt{2}}{2}</td>
</tr>
<tr>
<td>π/2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>3π/4</td>
<td>-\frac{\sqrt{2}}{2}</td>
<td>\frac{\sqrt{2}}{2}</td>
</tr>
<tr>
<td>π</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>5π/4</td>
<td>-\frac{\sqrt{2}}{2}</td>
<td>-\frac{\sqrt{2}}{2}</td>
</tr>
<tr>
<td>3π/2</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>7π/4</td>
<td>\frac{\sqrt{2}}{2}</td>
<td>-\frac{\sqrt{2}}{2}</td>
</tr>
</tbody>
</table>

You have 110 minutes to finish your exam. The exam is worth 50 points total.

Questions

1. Define the following terms:
   (a) (2 points) the partial derivative of a function \( f \) of \( n \) variables with respect to a variable \( x_i \) at the point \((x_1, x_2, \ldots, x_n)\).
   (b) (2 points) an \( n \)-dimensional vector.
   (c) (2 points) the dot product of two \( n \)-dimensional vectors \( \vec{v} \) and \( \vec{w} \).

2. For each of the following decide if it is a function or not. If it is not, give a brief explanation for why it is not.
   (a) (2 points) The amount of money you owe on a loan depending on the principal amount, the interest rate, and the time since the loan was made (assuming no payments have been made).
   (b) (2 points) \( f(x, y) = \pm \sqrt{x^2 + y^2} \).
   (c) (2 points) The average height of a student enrolled in a UCSD course depending on the heights of every enrolled student and the total number of enrolled students.
   (d) (2 points) The temperature depending on latitude and longitude.
   (e) (2 points) \( g(x, y, z) = x \cos(y) + \log(z^2 + y^2 + 1) + e^{xy} \).
3. Mark each of the following as “true” or “false”

(a) (1 point) If \( \vec{u} \) is a nonzero 3-dimensional vector and \( D_\vec{v}(f)(1, 3, 7) \) is positive, then if \( \vec{u} = -2\vec{v} \), \( D_{-2\vec{u}}(f)(1, 3, 7) \) is negative.

(b) (1 point) The vectors \( \vec{v} = (2, 3) \) and \( \vec{w} = (-9, 6) \) are perpendicular.

(c) (1 point) There are infinitely many vectors in the same direction as \( \vec{v} = (1, 7) \).

(d) (1 point) The gradient of a function of five variables at a point \((x_1, x_2, x_3, x_4, x_5)\) is a 5-dimensional vector.

(e) (1 point) If \( f \) is a function of the variables \( x \) and \( y \) and \( \frac{df}{dx}(1, 2) > 0 \), then \( f \) is increasing in the negative \( x \) direction at \((1, 2)\).

(f) (1 point) There are infinitely many vectors parallel to \( \vec{v} = (1, 7) \).

(g) (1 point) The magnitude of \( \vec{V}(f)(1, 2, 3) \) is the minimum value of \( D_{\vec{v}}(f)(1, 2, 3) \) for any nonzero vector \( \vec{v} \).

(h) (1 point) If \( g \) is a function of two variables and \( g_x(-1, 2) > 0 \), then \( g \) must be increasing at \((-1, 2)\) in the direction of any vector with a positive \( x \)-coordinate.

4. Let \( \vec{v} = (3, \pi, e^2, 2, 7) \) and \( \vec{w} = (1, 0, e^{-2}, -2, 0) \).

(a) (2 points) What is \( \vec{v} \cdot \vec{w} \)?

(b) (2 points) What is the angle between \( \vec{v} \) and \( \vec{w} \)?

(c) (2 points) What is \( 2\vec{v} - 6\vec{w} \)?

5. Let

\[
h(x_1, x_2, x_3, x_4, x_5, x_6) = \cos(x_3)\sin(x_1) + x_2^2e^{x_1x_5} + x_6^2\ln(x_1^2 + 1).
\]

(a) (6 points) Compute all (first-order) partial derivatives of \( h \).

(b) (4 points) What is \( D_{\vec{v}}(h)(\pi/2, 4, \pi, 0, 7, 0) \), where \( \vec{v} = (0, 1, 0, 0, 0, 0) \)?

6. Simon cautiously walks through the radioactive crater of a meteor that recently struck Earth. The meteor lies 8 meters north and 6 meters east of his current location. Using his Geiger counter, Simon estimates that the partial derivatives of the radiation levels are

\[
\frac{\partial R}{\partial x} = 5, \quad \frac{\partial R}{\partial y} = 8,
\]

where north is the positive \( y \)-direction and east is the positive \( x \)-direction.

(a) (2 points) What is the vector that represents the direction from Simon to the meteor?

(b) (4 points) If Simon heads directly toward the meteor, will he experience higher radiation levels? How do you know?

(c) (4 points) Simon wants to be cautious, but he’s still curious about the meteor. He decides to head in the direction \((4, 3)\). Is this more or less dangerous than walking directly toward the meteor? How do you know?