Math 103A Fall 2005 HW 8 (last one!)

HW Due Friday 12/2/05 in class

All exercise and page numbers refer to Gallian, 6th edition.

0. These exercises are suggestions for extra practice at home (or in section) and are not to be turned in!

Gallian Chapter 10, #9, 15, 21, 25, 35

Gallian Chapter 11, #5, 9, 13, 17, 19

1. Gallian Chapter 10, #4, 18, 20, 30, 32

2. Gallian Chapter 11, #2, 12, 20, 28, 32

Hint for #20: The question should read “Suppose that $G$ is a finite Abelian group that has exactly one subgroup $H$ with $|H| = d$ for each divisor $d$ of $|G|$. Show that $G$ is cyclic.” To approach this problem, I suggest the following proof of the contrapositive. Think about Corollary 1 and 2 in Chapter 8 on page 156. Use these to show that if $G$ is not cyclic, then $G \cong \mathbb{Z}_{a_1} \oplus \mathbb{Z}_{a_2} \oplus \cdots \oplus \mathbb{Z}_{a_n}$ where $\gcd(a_i, a_j) = d > 1$ for some pair $a_i, a_j$. Then show that the group $\mathbb{Z}_{a_i} \oplus \mathbb{Z}_{a_j}$ has more then one element of order $d$. Use this to find more than one element of order $d$ in $G$.

Hint for #32: Calculate the order of $G$ and the order of each of the elements in $G$. The question “What is the isomorphism class of $G$” means find positive numbers $a_i$ such that $G \cong \mathbb{Z}_{a_1} \oplus \mathbb{Z}_{a_2} \oplus \cdots \oplus \mathbb{Z}_{a_n}$.