Section 5.2 Problem 12

We first calculate the coefficients \( c_n \) of the exponential series for \( e^x \). They are given by the following formula (observe that in the previously posted solution the factor in front of the integral was wrong):

\[
c_n = \frac{1}{2\ell} \int_{-\ell}^{\ell} e^x e^{-i\frac{n\pi}{\ell}x} \, dx =
\]

\[
= \frac{1}{2\ell(1 - i\frac{n\pi}{\ell})} (e^{\ell - in\pi} - e^{in\pi - \ell}) =
\]

\[
= \frac{(-1)^n(\ell + in\pi)}{\ell^2 + n^2\pi^2} \sinh(\ell),
\]

since \( e^{in\pi} = (-1)^n \) and \( \sinh(x) = \frac{1}{2}(e^x - e^{-x}) \). Similarly, we obtain for the coefficients \( \tilde{c}_n \) of the exponential series for \( e^{-x} \) that

\[
\tilde{c}_n = \frac{(-1)^n(\ell - in\pi)}{\ell^2 + n^2\pi^2}.
\]

As \( \cosh x = \frac{1}{2}(e^x + e^{-x}) \), we obtain

\[
\cosh x = \sum_n \frac{(-1)^n\ell}{\ell^2 + n^2\pi^2} e^{n\pi ix/\ell}.
\]