Math 3C
Practice Final

1. Consider the line \( y = 5x + 3 \).
   
   (a) (2 points) What is the slope and y-intercept of this line?
   
   (b) (3 points) Find the equation of the line through \((2, 1)\) that is perpendicular to the line above. Write your final answer in \( y = mx + b \) form.
   
   (c) (2 points) Let \( \theta \) be the angle that the line \( y = 5x + 3 \) makes with the \( x \)-axis. Find \( \tan(\theta) \).

2. Consider the quadratic equation \( f(x) = x^2 + 4x + 1 \).
   
   (a) (3 points) What is the minimum value of the function above?
   
   (b) (1 point) At what \( x \) value does the minimum of \( x^2 + 4x + 1 \) occur?
   
   (c) (1 point) What is the minimum value of \( e^{x^2+4x+1} \).
   
   (d) (1 point) At what \( x \) value does the minimum value of \( e^{x^2+4x+1} \) occur?

3. Let \( f(x) = (x - 3)^2 - 2 \).
   
   (a) (3 points) Sketch the function, indicate the location of the vertex and two additional points.
   
   (b) (2 points) Expand \( f(x) \) into \( ax^2 + bx + c \) form.
   
   (c) (2 points) Solve \( f(x) = 0 \).
   
   (d) (2 points) Restrict \( f \) to the domain \([3, \infty)\). Find \( f^{-1} \).

4. (6 points) Solve the system:
   
   \[
   \begin{cases}
   x^2 + \ln y = 6 \\
   2x^2 - 3 \ln y = 2
   \end{cases}
   \]
5. Consider the following rational functions:

\[ f(x) = \frac{x^3 + 1}{x^2 + 1} \quad g(x) = \frac{3x}{x + 1} \]

(a) (3 points) What is \( f(x) + g(x) \)?

(b) (2 points) Describe the end behavior of \( g(x) \)? (i.e. What happens to \( g(x) \) as \( x \to \infty \) and \( x \to -\infty \)?)

(c) (2 points) Describe all the asymptotes of \( g(x) \) (both horizontal and vertical). If there is no vertical or horizontal asymptote, write that and say why.

6. Recall that bacteria colonies in a petri dish grows exponentially and thus the number of bacteria in the dish at time \( t \) can be modeled by the equation \( f(t) = c \cdot b^t \). In a given petri dish, there are 10 bacteria cells at the start of the day. After 1 hour, there are 20 bacteria cells.

(a) (2 points) What two conditions are satisfied by \( f(t) \).

(b) (3 points) Solve for \( c \) and \( b \).

(c) (2 points) How many bacteria will there be after 4 hours? Simplify your answer.

(d) (2 points) When will the population reach 100 bacteria cells?

7. Suppose Sally wants to know the height of the Sun God statue on campus. On a bright sunny day, Sally, who’s 6 feet tall, determines her shadow is 9 feet long and Sun God’s shadow is 45 feet long. See the (super awesome) figure below.

(a) (2 points) Use Sally’s height and shadow length to find an expression for \( \theta \), (i.e. \( \theta = \ldots \)).

(b) (3 points) Find the height of the Sun God Statue. Simplify as much as possible.

8. Simplify the following as much as possible.

(a) (4 points) Assume \( \sin(\theta) = 1/5 \) and \( 0 \leq \theta \leq \pi/2 \), find \( \sin(-\theta) \), \( \cos(\theta) \) and \( \csc(\theta) \).

(b) (1 point) \( \sin \left( \sin^{-1}(1/2) \right) \)

(c) (2 points) \( \cos^{-1} \left( \cos(3\pi) \right) \)

(d) (3 points) \( \cos \left( \sin^{-1}(2/5) \right) \)