Math 181A Worksheet Week 1

Useful Fact:

• If \( f \) is recognized to be a density function, then \( \int_{\mathbb{R}} f(x)dx = 1 \).
• Moreover, by invariance of translation \( \int_{\mathbb{R}} f(x+b)dx = 1 \).
• With a linear substitution, \( \int_{\mathbb{R}} f(kx+b)dx = \frac{1}{k} \) follows.

Examples:

1. Marginal through independence Let \( f \) and \( g \) be two density functions on \( \mathbb{R} \). Suppose the joint density \( f_{X,Y}(x,y) = f(x)g(y) \), show that the marginal density of \( X \) is \( f \).

2. Marginal through factorization Let \( f \) be a density function while \( g \) an arbitrary function. Suppose the joint density can be factorized as \( f_{X,Y}(x,y) = f(ax+by+c)g(x) \), show that the marginal density of \( X \) is \( \frac{1}{b}g(x) \).

Practice:

1. Find \( f_Y(y) \) if \( f_{X,Y}(x,y) = 2e^{-x}e^{-y} \) for region \( 0 \leq x \leq y \).

2. Find \( f_Y(y) \) if \( f_{X,Y}(x,y) = e^{-x}e^{-y} \) for region \( 0 \leq x, y \).

3. Find \( f_Y(y) \) if \( f_{X,Y}(x,y) = \frac{1}{\pi}e^{-2x^2-2xy-y^2} \) for region \( x, y \in \mathbb{R} \).