Math 181A Worksheet Week 10

Concept: Generalized Likelihood Ratio Test (Maximum Likelihood Ratio Test):

\[ H_0 : \theta \in \Theta_0 \] versus \[ H_1 : \theta \in \Theta_1 \]

- Find the MLE \( \hat{\theta}_0 = \arg \max_{\theta \in \Theta_0} L(\theta) \) and \( \hat{\theta}_1 = \arg \max_{\theta \in \Theta_1} L(\theta) \).
- Compute the test statistic \( \lambda = \frac{L(\hat{\theta}_0)}{L(\hat{\theta}_1)} \).
- Reject \( H_0 \) for small observed \( \lambda \)
  - If the exact distribution of \( \lambda \) (or certain monotonic transformation of it) is known, find the cutoff by its quantiles.
  - If the model is nested, i.e. \( \Theta_0 \) is a subspace of \( \Theta_0 \cup \Theta_1 \), use the asymptotic distribution \( -2 \log \lambda \overset{d}{\to} \chi^2_{df} \), \( df = \dim(\Theta_0 \cup \Theta_1) - \dim(\Theta_0) \).

Practice: Test the Following Hypothesis by GLRT

You may write your answer in quantile.

1. Simple vs Simple: \( X_1, \ldots, X_n \overset{iid}{\sim} \text{Poisson}(\lambda) \), \( \alpha = 0.05 \)
   \[ H_0 : \lambda = 1 \text{ versus } H_1 : \lambda = 2. \]

2. Nested Simple vs Many: \( X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu, 1) \), \( \alpha = 0.05 \)
   \[ H_0 : \mu = 0 \text{ versus } H_1 : \mu \neq 0. \]

3. Many vs Many: \( X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu, 1) \), \( \alpha = 0.05 \)
   \[ H_0 : \mu \leq 0 \text{ versus } H_1 : \mu > 0. \]