Math 181A Worksheet Week 6

Examples:

1. Consistency by Definition
   \( X_1, \ldots, X_n \overset{iid}{\sim} \text{Uniform}(0, \theta) \), show that \( \hat{\theta} = X_{\text{max}} \) is a consistent estimator for \( \theta \).

2. Consistency by Mean Square Error
   Suppose \( \hat{\theta}_n \) is an estimator for \( \theta \) with sample size \( n \). If \( \lim_{n \to \infty} \mathbb{E} \hat{\theta}_n = \theta \) and \( \lim_{n \to \infty} \text{var} \hat{\theta}_n = 0 \), then \( \hat{\theta} \) is consistent. (Hint: Use the Tchebychev/Markov’s Inequality in the following form \( \mathbb{P}(|X - \mathbb{E}X| \geq \epsilon) \leq \frac{\text{var}(X)}{\epsilon^2} \))

Practice:

1. Suppose \( X_1, \ldots, X_n \overset{iid}{\sim} \text{Poisson}(\lambda) \), show that \( \hat{\lambda} = \bar{X} \) is a consistent estimator for \( \lambda \).

2. \( X_1, \ldots, X_n \overset{iid}{\sim} \mathcal{N}(\mu, \sigma^2) \), show that \( \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 \) is a consistent estimator for \( \sigma^2 \).

3. \( X_1, \ldots, X_n \overset{iid}{\sim} \text{Exp}(\theta) \), show that \( \hat{\theta} = X_1 \) is NOT a consistent estimator for \( \theta \).