Math 181A Worksheet Week 7

Homework Review: 5.7.1, 5.7.2

Concepts

1. **Confidence Interval**
   Describe the goodness of estimation: centered at estimate/estimator, use estimate/estimator for parameter when needed.

2. **Hypothesis Testing**
   Test certain hypothesis: centered at null hypothesis, use null hypothesis for parameter when needed

Practice:

Suppose $7, 6, 6, 10, 6 \overset{iid}{\sim} Binom(10, p)$.

1. Find the MLE for $p$.
   **Solution:**
   Likelihood:
   $$L(p) \propto p^{7+6+6+10+6}(1-p)^{50-7-6-6-10-6}$$
   Log-likelihood:
   $$l(p) = 35 \log p + 15 \log(1 - p) + C$$
   Find critical points:
   $$\frac{35}{p} = \frac{15}{1 - p} \Rightarrow p = 0.7$$
   Check boundary limits:
   $$l(0+) = l(1-) = -\infty$$
   Therefore,
   $$\hat{p}_{MLE} = 0.7$$

2. Construct a 95% confidence interval for $p$
   **Solution:**
   By Theorem 5.3.1 with $k = 35$ and $n = 50$,
   $$[0.7 - 1.96\sqrt{\frac{0.3 \times 0.7}{50}}, 0.7 + 1.96\sqrt{\frac{0.3 \times 0.7}{50}}]$$
3. Test the hypothesis at level 0.05

\[ H_0 : p = 0.5, H_\alpha : p \neq 0.5 \]

**Solution:**
The sample size is quite big, you may use Normal approximation.
The test interval is given by

\[
[0.5 - 1.96 \sqrt{\frac{0.5 \times 0.5}{50}}, \ 0.5 - 1.96 \sqrt{\frac{0.5 \times 0.5}{50}}] = [0.36, 0.64]
\]

Since 0.7 \notin [0.36, 0.64], \( H_0 \) is rejected in favor of \( H_\alpha \).

4. Test the hypothesis at level 0.025

\[ H_0 : \mu = 0.6, H_\alpha : \mu > 0.6 \]

The sample size is quite big, you may use Normal approximation.
The test interval is given by

\[
(-\infty, 0.6 + 1.96 \sqrt{\frac{0.6 \times 0.4}{50}}] = (-\infty, 0.74]
\]

Since 0.7 \in (-\infty, 0.74], we accept \( H_0 \).