Math 181B Worksheet Week 1

Likelihood Ratio Test: Simple vs Simple

Example:

\(X_1, \ldots, X_n \sim \text{Normal}(\mu, 1), \alpha = 0.05\)

\(H_0 : \mu = 0 \, \text{versus} \, H_1 : \mu = 1.\)

Review: Maximum Likelihood Estimator

Example:

Let our sample be 1.44, 6.08 from \(\text{Exp}(\theta)\). In your distribution sheet, the density is

\[ f_\theta(x) = \frac{1}{\theta} e^{-x/\theta} \quad x > 0 \]

Find the Maximum Likelihood estimator for \(\theta\) and compute its estimate.

Practice:

1. **Likelihood Ratio Test**: \(X_1, \ldots, X_n \sim \text{Poisson}(\lambda), \alpha = 0.05\)
   Derive the likelihood ratio test for \(H_0 : \lambda = 1 \, \text{versus} \, H_1 : \lambda = 2.\) You may leave your result in terms of quantile. (Hint: independent sum \(\text{Poisson}(\lambda_1) + \text{Poisson}(\lambda_2) = \text{Poisson}(\lambda_1 + \lambda_2)\))

2. **Two Sample MLE** Let \(X_1, X_2 \sim \mathcal{N}(\mu_X, 1)\) and \(Y_1, Y_2, Y_3 \sim \mathcal{N}(\mu_Y, 1)\). In your distribution sheet, the p.d.f. for \(\mathcal{N}(\mu, \sigma^2)\) is

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad x \in \mathbb{R} \]

Consider the following two cases.

- Assume \(\mu_X = \mu_Y = \mu\).

- Do not assume \(\mu_X = \mu_Y\).