PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. Give the definition of:
   (i) a left coset.
   (ii) the index of a subgroup.
   (iii) the Cartesian product of two sets.
   (iv) the direct product of two groups.
   (v) an isometry of the plane.
   (vi) a group homomorphism.
   (vii) image of a subgroup.
   (viii) inverse image of a subgroup.
   (ix) the kernel.
   (x) a normal subgroup.
   (xi) a factor group.
   (xii) automorphism of a group.

2. If \( H \times G \) is the direct product of two groups \( H \) and \( G \), \( h \in H \) has order \( m \) and \( g \in G \) has order \( n \) then what is the order of \( (h, g) \in H \times G \)?

3. State the fundamental theorem of finitely generated abelian groups. Find all abelian groups of order 216.

4. If \( \phi : \mathbb{Z} \longrightarrow S_7 \) is a group homomorphism such that
   \[ \phi(1) = (1,7)(1,6)(1,5)(1,4)(1,3)(1,2) \]
   then what is \( \phi(100) \)?

5. Find a homomorphism \( S_4 \longrightarrow \mathbb{Z}_2 \) which is onto.

6. Is there a homomorphism \( S_4 \longrightarrow \mathbb{Z}_5 \) which is onto? Explain.