PRACTICE PROBLEMS FOR THE FIRST MIDTERM

1. Give the definition of:
   (i) a ring.
   (ii) a commutative ring.
   (iii) a zero divisor.
   (iv) an integral domain.
   (v) the characteristic of a ring.
   (vi) an ideal.
   (vii) the quotient of a ring by an ideal.
   (viii) a prime ideal.
   (ix) a maximal ideal.
   (x) the field of fractions of an integral domain.

2. Let $R$ be a ring and let $I$ and $J$ be two ideals.
   (i) Show that the intersection $I \cap J$ is an ideal.
   (ii) Is the union $I \cup J$ an ideal?

3. Let $R$ be an integral domain and let $I$ be an ideal. Show that $R/I$ is a field if and only if $I$ is a maximal ideal.

4. (i) Let $R$ be an integral domain. If $ab = ac$, for $a \neq 0$, $b, c \in R$, then show that $b = c$.
   (ii) Show that every finite integral domain is a field.

5. If
   
   $I_1 \subset I_2 \subset I_3 \subset \cdots \subset I_n \subset \cdots$

   is an ascending sequence of ideals in a ring $R$ then the union $I$ is an ideal.

6. Let $R$ be a ring and let $S = M_{2,2}(R)$ be the ring of all $2 \times 2$ matrices with entries in $R$. If $I$ is an ideal of $S$ then show that there is an ideal $J$ of $R$ such that $I$ consists of all $2 \times 2$ matrices with entries in $J$. 

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