Question 1.

A walk on the integers starts at zero and then at each step moves one unit to the left or one unit to the right. Determine (a) the number of walks with $k$ steps (b) the number of walks with $k$ steps which end at zero and (c) the number of walks of $k$ steps whose last point is a negative number (such as $0 \rightarrow 1 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow -1 \rightarrow -2$).
Question 2.
Prove that for any positive integer $n$,

\[ \binom{n - \frac{1}{2}}{n} = \frac{1}{4^n} \binom{2n}{n}. \]
Question 3.

State the Binomial Theorem precisely. Then determine the exact value of

\[ \sum_{k=1}^{\infty} \binom{-\frac{1}{2}}{k} 2^k. \]
Question 4.
Let $k, n$ be positive integers where $n \geq k$. Let $a_n$ be the number of ways of writing $n$ as a sum of $k$ integers, where each of the integers is either one or two. For instance, there are three ways to write five as a sum of three ones and twos, namely:

$$5 = 1 + 2 + 2 = 2 + 2 + 1 = 2 + 1 + 2.$$ 

Determine the generating function $a_n$, and use the generating function to determine a formula for $a_n$ for all positive integers $n$. Then determine the number of ways of writing nine as a sum of seven ones and twos.